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THE JOURNAL
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Volume VI Number 4

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The Absorption of Sound in Gases

VERN O. KNUDSEN, *University of California at Los Angeles**

(Received February 4, 1935)

RECENT theoretical and experimental work by a uumber of physicists has shown that the measurement of the velocity and absorption of sound in gases provides a new technique for investigating molecular collisions. The classical theories of Stokes, Kirchhoff and Rayleigh on the absorption of sound in gases were based on the effects of viscosity and heat conductivity, and until recently it has been assumed generally that the theories based on these effects accounted satisfactorily for the observed attenuation of sound in the air and in other gases.

Jeans,[1] in 1904 showed that in all polyatomic gases, that is, gases for which the ratio of the specific heats is less than five-thirds, there would be complications affecting the velocity and absorption of sound not anticipated by the usual theories unless the transfers of energy between translational and rotational (or vibrational) degrees of freedom occur with sufficient rapidity to keep pace with the changes of translational energy. He assumed molecules of the "loaded sphere" type, and set up four equations, namely, the continuity equation, the force equation, an equation for the transfer of translational energy, and one for the transfer of rotational energy. His solution of these equations shows: (1) That the velocity of sound c in a gas of this assumed type has the limiting values $(7/5\cdot p/\rho)^{\frac{1}{2}}$ and $(5/3\cdot p/\rho)^{\frac{1}{2}}$ (where p is the pressure and ρ the density of the gas), the latter value applying to the case where the sound frequency is high and the rate of energy transfer is relatively low; and (2) that the slowness of the rate of energy transfer will manifest itself by an increased (anomalous) absorption. In the case Jeans considered, however, the increase in absorption was relatively small—only about one-seventh as large as the absorption attributable to viscosity. Consequently, no serious consideration has been given to this early work of Jeans until recently.

In 1928, Herzfeld and Rice,[2] by means of more general but similar equations, including a time constant τ, the reciprocal of which measures the rate of exchange between external (translational) and internal (vibrational or rotational) energy, obtained dispersion and absorption formulas

(* This paper was presented by Professor Knudsen, December 29, 1934, in connection with the general meeting of the American Association for the Advancement of Science, and was awarded the annual $1000 A.A.A.S. Prize as the noteworthy paper of the meeting. EDITOR.)

[1] Jeans, *Dynamical Theory of Gases*, 2nd ed., p. 374.

[2] K. F. Herzfeld and F. O. Rice, Phys. Rev. **31**, 691–695 (1928).

which offered an explanation for the dispersion in CO_2 which had been observed by Pierce,[3] at supersonic frequencies, and a possible explanation for the high absorption in air which had been observed by Naklepajew,[4] also at supersonic frequencies. Similar theoretical studies have been made by Bourgin,[5] Kneser,[6] Rutgers,[7] Henry[8] and Richards.[9] The work of Kneser, the theoretical part of which was modeled after a treatise by Einstein[10] on the propagation of sound in a partially dissociated gas, not only led to convenient equations for calculating the velocity and absorption of sound in a dispersive gas—essentially the same equations as the ones that had been obtained previously by Herzfeld and Rice—but was followed by an experimental study[11] on the velocity of sound in CO_2 which gave results in good agreement with his dispersion formula.

The essential features of this modern theory of sound dispersion in gases, in its simplest form, was recently described in this journal by Kneser.[12] It is summarized here as follows: In a polyatomic gas there will be in general a certain number of vibrating or rotating molecules, the number being a function of temperature. A change of temperature will be followed by a change in the number of "excited" (vibrating or rotating) molecules. Such temperature changes occur when a sound wave passes through a gas so that there will always be a characteristic time lag between the temperature change (or change of density) and the molecular transitions (transfers between translational energy of molecules and either vibrational or rotational energy) for each possible kind of vibration or rotation of the molecules. Since the pressure depends only on the translational energy there will be a lag of pressure p behind the density ρ, so that the velocity of sound c which is ordinarily given by $c^2=\delta p/\delta\rho$ must be modified to include the phase lag. That is,

$$c^2=(\delta p_0/\delta\rho_0)e^{j\phi}, \tag{1}$$

where δp_0 and $\delta\rho_0$ are the maximal absolute values of the change of pressure and the change of density, ϕ is the phase angle by which the pressure lags behind the density, and j is $(-1)^{\frac{1}{2}}$. Assuming an ideal gas, $\rho V=$constant and $pV=RT$, where V is the volume, T the temperature, and R the gas constant,

$$c^2=(p/\rho)[1-R\delta T/p\delta V]. \tag{2}$$

In the simplest form of theory such as is here sufficient to account for the observed absorption in air and oxygen, it is assumed that there are only normal nonvibrating molecules, and one kind of vibrating molecules, with a frequency ν, and a characteristic "lifetime" τ, and that the transitions between the n_1 excited and n_0 normal molecules are governed by

$$-\partial n_1/\partial t=k_{10}n_1-k_{01}n_0, \tag{3}$$

where k_{10} and k_{01} are reaction rate constants. Thus, k_{10} gives the number of transitions from the excited to the normal state per molecule per second, and k_{01} gives the number of transitions in the reverse direction. (The reciprocal of k_{10} is approximately equal to the time constant τ.) At ordinary temperatures $n_1\ll n_0$ and therefore $k_{01}\ll k_{10}$. By variation of (3), replacing $\partial/\partial t$ by the operator $j\omega$ (where ω is 2π times the frequency of the sound wave), applying the Boltzmann law $n_1=n_0e^{-E/RT}$ (where E is the vibrational energy per mole), and putting $-p\delta V=C_\infty\delta T+h\nu\delta n_1$ (where C_∞ is the molecular heat at infinite sound frequency, that is, it is that portion of the heat capacity which is independent of frequency, and h is Planck's constant), (2) becomes

$$c^2=\frac{p}{\rho}\left\{1+\frac{R}{\left[C_\infty+\frac{1+k}{k}(C_0-C_\infty)\frac{k}{(1+k+j\omega/k_{10})}\right]}\right\}, \tag{4}$$

where C_0 is the molecular heat at zero frequency, and $k=k_{01}/k_{10}$. The real part of (4) gives the

[3] G. W. Pierce, Proc. Nat. Acad. Sci. **60**, 271–302 (1925).

[4] N. Naklepajew, Ann. d. Physik **35**, 175–181 (1911).

[5] D. J. Bourgin, Phil. Mag. **7**, 821–841 (1929); Phys. Rev. **34**, 521–526 (1929); **42**, 721–730 (1932); J. Acous. Soc. Am. **4**, 108–111 (1932).

[6] H. O. Kneser, Ann. d. Physik **11**, 761–776 (1931); **16**, 360–361 (1933); J. Acous. Soc. Am. **5**, 122–126 (1933).

[7] A. J. Rutgers, Ann. d. Physik **16**, 350–359 (1933).

[8] P. S. H. Henry, Proc. Camb. Phil. Soc. **28**, 249–255 (1931–32).

[9] W. T. Richards, J. Chem. Phys. **1**, 863–879 (1933).

[10] A. Einstein, Sitz.-Ber. Berlin Acad. Math. Kl. 380–385 (1920).

[11] H. O. Kneser, Ann. d. Physik **11**, 777–801 (1931).

[12] H. O. Kneser, J. Acous. Soc. Am. **5**, 122–126 (1933).

phase velocity, and the ratio of the imaginary to the real parts gives tan ϕ, from which the absorption coefficient, in which we are primarily interested, can be readily evaluated. The absorption coefficient per wavelength becomes

$$\mu = 2\pi\left[\frac{RC_i}{C_\infty(C_\infty+R)}\cdot\frac{\omega k_{10}}{k_{10}^2+\omega^2}\right], \qquad (5)$$

where $C_i = C_0 - C_\infty$ is the internal heat capacity. The maximal value of the absorption coefficient μ_m occurs when $k_{10} = \omega$, so that

$$\mu_m = \pi[RC_i/C_\infty(C_\infty+R)]. \qquad (6)$$

From (6) it is seen that the maximal absorption, which occurs when $\omega = k_{10}$, is a constant which, for a certain gas, depends only on C_i and C_∞, and that the maximal absorption coefficient per unit length, $m_m = \mu_m/\lambda$ (where λ is the wavelength), is proportional directly to the frequency, and not to the square of the frequency, as is the case for classical absorption. These conclusions are well confirmed by the experimental findings in air and O_2. Further, it is evident from (5) and (6) that it is possible to determine the reaction constant k_{10} either for a pure gas or a gas mixture simply by determining the frequency ν_m at which the absorption is a maximum, since under this condition $k_{10} = \omega = 2\pi\nu_m$.

In order to illustrate the manner of using (5) and (6) for the calculation of μ or μ_m for a gas such as O_2, for which the vibrational energy E (4420 cal. per mole) is well known, we shall write down the values of some of the quantities that are to be substituted in (6). Thus,

$$C_i = \frac{E^2}{RT^2}\cdot\frac{e^{-E/RT}}{1-e^{-E/RT}}, \quad e^{-E/RT} \ll 1, \quad \text{and} \quad C_\infty = \frac{5}{2}R,$$

and therefore $\mu_m = 104\times10^{-4}$ at 20°C. This value is well confirmed by our experiments with oxygen for frequencies up to 20,000 cycles; and similar experiments with oxygen at 55°C give confirmation of the temperature dependence required by Eqs. (5) and (6).

If the maximal absorption in O_2 occurs in the audible range of frequencies, as was conjectured Henry,[8] it should be possible to test the validity of (5) and (6) by measurements of the reverberation of sound in a room or the attenuation of sound in the free atmosphere.

As a matter of fact, before the above theory had been fully developed, the author's attempt to calibrate a new reverberation chamber[13] revealed that the absorption in the air at ordinary temperatures and humidities was much greater than the value predicted by the classical theory. Further, it was found that the surface absorption of the painted walls did not depend upon humidity or temperature. These results were obtained by measuring the rate of decay of sound in two similarly shaped rooms, having the same boundary material (painted concrete) but different "mean free paths" (the average distance the sound waves travel between successive reflections).

The success of the initial "two room" experiments suggested the desirability of using smaller chambers in which the temperature and humidity of the air could be more easily controlled, and in which other gases could be investigated. Accordingly, a 6-foot cubical chamber and a 2-foot cubical chamber were constructed from $\frac{1}{4}$-inch steel boiler plate, strengthened with angle irons two feet on centers. The chambers were equipped with rotating paddles which helped to maintain a diffuse state of sound. This apparatus, the theory on which the method of measurement is based, and preliminary results obtained from

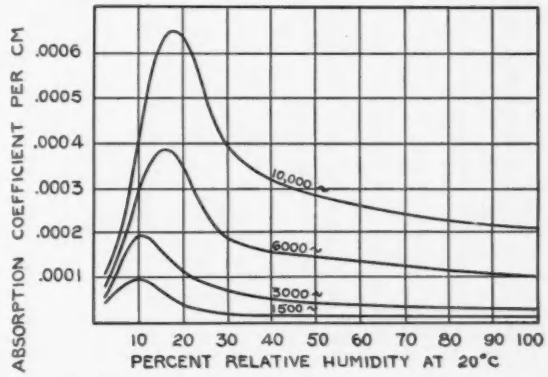

FIG. 1. Curves showing absorption coefficients in air at 20°C and at different relative humidities for frequencies of 1500, 3000, 6000 and 10,000 cycles.

[13] V. O. Knudsen, J. Acous. Soc. Am. **3**, 126–138 (1931). (P. E. Sabine, J. Frank. Inst. **207** (3), 341–368, March (1929), and E. Meyer, Zeits. f. tech. Physik **7**, 253 (1930), had previously found that reverberation times at high frequencies depended upon the humidity, but their experiments, conducted in a single room, did not reveal whether the phenomenon was a "surface" or a "medium" effect.)

measurements in oxygen, nitrogen and air, containing small amounts of water vapor, were described in this journal in 1933.[14]

Fig. 1 shows our most recent results from a series of measurements on the absorption of audible sound in air of different relative humidities at a temperature of 20°C, for frequencies of 1500, 3000, 6000 and 10,000 cycles. At each frequency, as was shown in the earlier paper,[14] there is a certain humidity at which the absorption is a maximum. Further, the magnitudes of these maxima are proportional directly to the frequency, and not the square of the frequency, and the coefficients of absorption at the maxima are as much as 100 times as large as the coefficients predicted by the classical theories.

Experiments in O_2 and N_2 revealed that this anomalous absorption was entirely attributable to the O_2, and in fact Kneser, who was then at the University of California, showed that our results for both air and O_2 were in good agreement with (5) and (6) provided the absorption resulted from energy transfers between O_2 molecules, and provided further that the reaction constant k_{10} were a quadratic function of the concentration of the water vapor.[15]

The good agreement between the observed and calculated values of the absorption of sound in O_2 plus H_2O indicated that similar absorption measurements in pure or mixed gases should provide an effective means for investigating energy transitions during molecular collision. Accordingly, Kneser and the author undertook a series of experiments on the absorption of sound in O_2 containing known amounts of one of a number of gaseous impurities.[16] The results of such a series of absorption measurements at 3000 and 6000 cycles are shown in Fig. 2. The observed values of the absorption coefficients are indicated by small circles. The solid line curves are calculated from Eq. (5), with the added assumption that the reaction constant k_{10} is proportional to the concentration h of added impurity—an assumption which is based upon the experimental finding that for all impurities except H_2O the concentration at which the absorption is a maximum is proportional to frequency. It will be noted that the absorption maximum is the same for all impurities, and the magnitudes agree with the value required by Eq. (6) for O_2. However, the kind of impurity is seen to affect the position of the absorption maximum. Stated otherwise, the reaction constant k_{10} is strongly influenced by the presence of gaseous impurities, a conclusion reached by Richards and Reid[17] and by Eucken and Becker[18] from dispersion measurements. Of the impurities studied to date, C_2H_5OH is the most effective molecule for exciting and de-exciting the oxygen molecule.

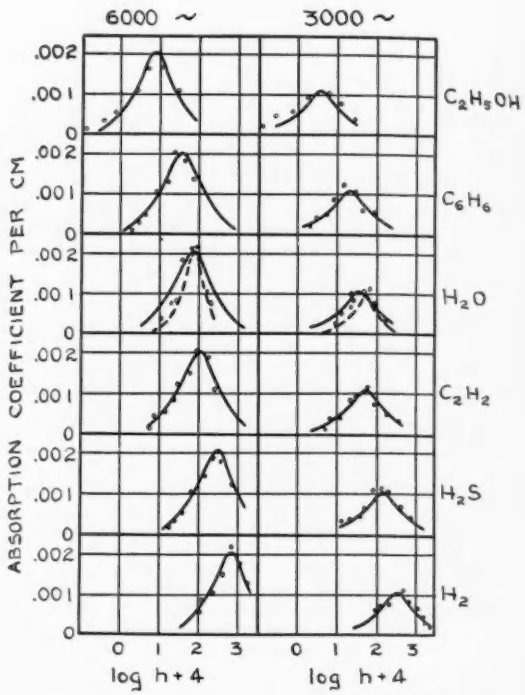

FIG. 2. Curves showing absorption coefficients in oxygen at 20°C containing small amounts of gaseous impurities. The curves at the left are at 6000 cycles, and those at the right 3000 cycles.

For all impurities except H_2O, the reaction constant appears to be proportional directly to the concentration,[19] that is,

[14] V. O. Knudsen, J. Acous. Soc. Am. **5**, 112–121 (1933).

[15] H. O. Kneser, J. Acous. Soc. Am. **5**, 122–126 (1933).

[16] Kneser and Knudsen, Ann. d. Physik **21**, 682–696 (1935).

[17] Richards and Reid, J. Chem. Phys. **2**, 206–214 (1934).

[18] Eucken and Becker, Zeits. f. physik. Chemie **B20**, 467–474 (1933).

[19] Recent experiments by Mr. Leonard Obert and the author in O_2 plus NH_3 indicate that the reaction constant k_{10} is proportional to the concentration of NH_3 over a range of frequencies from 1000 to 30,000 cycles.

$$k_{10}=ah \tag{7}$$

so that Eq. (5) becomes

$$\mu=2\pi\left[\frac{RC_i}{C_\infty(C_\infty+R)}\cdot\frac{\omega ah}{a^2h^2+\omega^2}\right], \tag{5'}$$

which is the formula used for calculating the absorption curves in Fig. 2. The value of a is calculated from the *one* value of k_{10} which, at the concentration for which the absorption is a maximum at a certain frequency (6000 cycles), is 2π times that frequency. This value of a is then used for calculating all other points on the $m-h$ curves at the two frequencies of 3000 and 6000 cycles.

Of the impurities worked with to date, only H_2O fails to follow the simple relation given by Eq. (7). In our earlier work it appeared that the reaction constant k_{10} was a quadratic function of the concentration h. Our most recent data indicate that the reaction constant involves a term proportional to the first power of h and also one proportional to the second power of h. For H_2O as an impurity in O_2 we have obtained the following empirical equation for the reaction constant

$$k_{10}=2.0\times10^6h-4.9\times10^8h^2. \tag{8}$$

The prominence of the second degree term in h would seem to indicate that a collision of an O_2 molecule with *two* H_2O molecules is much more efficient in producing a transition than is a collision between an O_2 molecule and *one* H_2O molecule. This is discussed at greater length in the paper by Kneser and the author[16] where Kneser offers the suggestion that this peculiar effect of water vapor may be attributed to the possibility of the chemical reaction, or at least the "affinity," suggested by $O_2+2H_2O\rightleftarrows2H_2O_2$. Further experiments are necessary, however, before this suggestion can be regarded as fact.

We can gain a better understanding of the foregoing results by calculating the collision frequencies and the probabilities of transitions in the different gas mixtures. Kneser has done this in the paper referred to above.[16] Thus, let W_{O_2} be the probability that a collision between a vibrating O_2 molecule and another O_2 molecule will result in a transition, and let W_F be the probability that a collision between a vibrating O_2 molecule and a foreign gas molecule will result in a transition; and let Z_{O_2} and Z_F be the number of corresponding collisions per molecule per second at room temperature and atmospheric pressure. Then the number of transitions per molecule per second from vibrating to normal states, that is k_{10} will be

$$k_{10}=W_{O_2}Z_{O_2}+W_FZ_F. \tag{9}$$

For pure O_2, $Z_F=0$, or $k_{10}=W_{O_2}Z_{O_2}<1000$ sec.$^{-1}$. From kinetic theory we obtain an approximate value for Z_{O_2}, namely, 4.2×10^9, so that $W_{O_2}<0.24\times10^{-6}$.

But in all cases of O_2 plus impurities we have investigated, $k_{10}\gg1000$. Hence, $W_{O_2}Z_{O_2}\ll W_FZ_F$, or (9) becomes for the mixtures we have studied

$$k_{10}=W_FZ_F. \tag{10}$$

From the Geiger and Scheel *Handbuch der Physik* (**9**, 399) it can be shown that for $h\ll1$,

$$Z_F=\frac{1}{4\sqrt{2}}hZ_{O_2}\left(1+\frac{m_{O_2}}{m_F}\right)^{\frac{1}{2}}\left(1+\frac{\sigma_F}{\sigma_{O_2}}\right)^2,$$

in which m is the mass, and σ the diameter of the molecules. Then, since $k_{10}=ah$ (for all impurities except H_2O)

$$W_F=\frac{k_{10}}{Z_F}=4\sqrt{2}a\Big/Z_{O_2}\left(1+\frac{m_{O_2}}{m_F}\right)^{\frac{1}{2}}\left(1+\frac{\sigma_F}{\sigma_{O_2}}\right)^2.$$

The quantity W_F, which gives the probability that the vibrating O_2 molecule will lose its vibrational energy upon colliding with a foreign molecule, gives useful information concerning the character of the foreign molecule.

TABLE I.

	Molecular Weight	Dipole Moment	$\frac{\sigma_F}{\sigma_{O_2}}$	"Affinity"	$a\times10^{-5}$	$W_F\times10^4$
C_2H_5OH	46	1.7	1.7	327	610	85
NH_3	17	1.44	1.2	90.7	188	26
C_6H_6	78	0	1.37	782	130	25
H_2O	18	1.9	0.91	——	20	——
					(4.9×10^8)*	——
C_2H_2	26	0	1.2?	311	43	8.3
H_2S	34	1.0	1.3	137	13	2.4
CCl_4	152	0	1.3	37.8	~10	~2.4
$HCCl_3$	118	1.2	2.0	70.5	9.6?	~1.3?
CO	28	0.1	1.1	68	~6	~1.3
H_2	2	0	0.83	68.4	4.8	0.5
CO_2	44	0	1.1		~1.7	~0.4
O_3	48	0	2?		~2	~0.3?
N_2	28	0	1.1	——	~0.5	~0.1
He	4	0	0.66		~0.4	~0.06
O_2	32	0	1		——	<0.2

* The coefficient of h^2 in Eq. (8).

In Table I are given the W_F values, together with other apparently pertinent data, for the molecules investigated in these experiments.

A complete explanation of the collision process does not seem possible from the few data as yet available. A number of tentative assumptions seem warranted, however:

1. The simple impulse theory of the transfer of energy at collision is inadequate since H_2, for example, is more effective than He.

2. If the colliding partner be a dipole the collision is more likely to disturb the nuclear vibrations of the O_2 molecule than if it be not a dipole. Thus, CO is more effective than the similar N_2 molecule.

3. One might suspect a possibility of disturbance when there is resonance, or near resonance, between the characteristic frequencies of the colliding pair. However, the findings do not support this possibility.

4. Franck and Eucken claim that the disturbance of the potential energy curves of the colliding molecules is the most important factor affecting the transfers between translational and vibrational energy. When certain molecules collide the binding energy of the nucleus is altered, and the nucleus will be set into vibration. It is not possible from our data to state in which cases this is predominant, but we can concur with the suggestion of Eucken and Becker[18] that a strong disturbance should be expected if the collision partners have a high chemical affinity.

In order to investigate further the dependence of the reaction constant k_{10} on the concentration of H_2O and other gaseous impurities in O_2, and to extend the measurements to higher frequencies and other gases, Mr. Obert and the author have modified our technique of measurement so that it is now possible to obtain absorption measurements for frequencies up to 40,000 cycles. The results of our investigation with this new technique will be described in a later communication to this journal.

In conclusion, the experiments and theory which have been described in this paper indicate that the measurement of sound absorption in gases provides a new approach to a number of important problems related to molecular collision. This new development is exhibited not only by the experiments described in this paper but also by work already completed or in progress on the dispersion and absorption of sound in gases, principally at supersonic frequencies, by Eucken, Becker and others at Göttingen, Richards and Reid at Princeton, Kneser and his students at Marburg, and Pielemeier and his students at Pennsylvania State. In addition, the absorption measurements in air are of general interest to every one concerned with atmospheric acoustics. They not only clarify a large number of curious problems associated with the influence of the weather on the acoustics of the atmosphere, but they have an immediate and practical application in architectural acoustics and sound signaling.

The author wishes to acknowledge again the important contribution Dr. Kneser has made to the interpretation of our experiments. His work with the author in the spring and summer of 1933, and his subsequent work at Marburg, have given a greatly enhanced significance to our experiments on the absorption of sound in gases.

The Electrical Phenomena of the Cochlea and the Auditory Nerve*

HALLOWELL DAVIS, *Department of Physiology, Harvard Medical School*

(Received February 16, 1935)

IN 1930, Wever and Bray[40, 41, 42] reported that, if electrical contact were made with the auditory nerve of a decerebrate cat and the potentials developed in it were amplified and delivered to telephone receivers, sounds delivered to the animal's ear could be recognized by a listener. Even words spoken to the animal could be understood and the speaker recognized by the quality of his voice. This observation immediately stimulated investigation of the electrical phenomena of the auditory mechanism, although it had already been shown by Buytendyk[8] and by Forbes, Miller and O'Connor[20] that electrical disturbances in response to sounds could be detected in the auditory nerve and medulla of the cat. Among the active workers in this field since 1930, we should mention Wever and Bray, Adrian, Crowe and Hughson, Culler, Hallpike and Rawdon-Smith, Bast, Guttman, and their various collaborators, in addition to our own group at Harvard. I shall attempt to summarize some of their results, particularly those which serve to illuminate the mechanism of sound reception in the ear. If I illustrate almost exclusively with our own material, it is because I am naturally most familiar with it.

I. Nature of Nervous Impulses

One reason for the intense interest aroused by the observations of Wever and Bray was that they appeared to show that frequencies of several thousand a second were transmitted over the auditory nerve to the brain. This supported the so-called "telephone theory of hearing," which assumes that the ear acts merely as a microphone or transformer and relays the audiofrequencies as nerve impulses to the brain, where they are supposed to be analyzed. The transmission of frequencies of several thousand per second implies a difference between the fibers of the auditory nerve and those of other nerves of the body, whose fibers cannot transmit nerve impulses at frequencies above approximately 1000 per second. This limitation of frequency of response in nerve fibers depends upon the nature of the nerve impulse. The latter is a wave of electrochemical change which sweeps over the fiber as a flame burns a gunpowder fuse, and it leaves the fiber refractory and incapable of conducting another impulse until a certain time for recovery has elapsed. The refractory period for fast mammalian fibers, structurally resembling those in the auditory nerve, is approximately 1 millisecond, setting a limit of 1000 per second for the highest frequency which can be transmitted. This limitation made the telephone theory appear most improbable from the physiological point of view.

II. Two Types of Response

Closer study of the Wever and Bray phenomenon showed that not *one* but *two* distinct types of electrical response are involved.[1, 3, 16, 25] Distinct from the nerve impulses which travel up the auditory nerve, another electrical disturbance is generated in the cochlea itself and is most readily detected from the round window. When this cochlear component is excluded by employing a suitable type of shielded electrode or by making contact with more remote portions of the auditory pathways in the substance of the brain, the very high frequencies are no longer recovered. The limiting frequency seems at present to be 3000, or possibly 4000, per second in the auditory nerve and 1000 in the tracts of the midbrain; whereas, from the auditory cortex of the brain, and even from the fibers leading immediately into it, it is impossible to recover any true audiofrequencies whatever.[35, 16, 18] Above the "limiting frequency," impulses in response to the sound are still present in the auditory nerve, but they are disorganized and not synchronized with the stimulating sound waves. The cochlear response, however, shows no such limitation of frequency. We have followed it in the cat up to 12,000 c.p.s. as a matter of routine study and find ourselves limited by our apparatus quite as much as by the animal at this point.

* This important paper was invited for presentation before the joint meeting of the Acoustical Society and the American Physical Society in Pittsburgh, December 29, 1934. EDITOR.

III. Difference between Cochlear Response and Action Potentials[16]

(1) The wave form of the cochlear response differs from that of the nerve. From the latter we recover a series of sharp transients having the wave form and the polarity characteristic of nerve impulses, while the cochlear response reproduces with considerable fidelity the wave form of the stimulating sound waves (Fig. 1). Even the

Fig. 1. Oscillograms of response to the tone of 462 c.p.s. *A*. Nerve impulses from eighth nerve by concentric electrodes. *B*. Cochlear response plus nerve impulses recorded from round window. The departure from sinusoidal wave form in *B* is due primarily to the presence of the nerve-impulse component.[18]

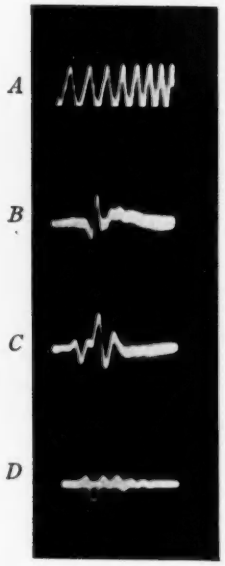

Fig. 2. Oscillograms of nerve impulses and cochlear response. *A*. 1000 c.p.s. wave to show time scale. *B*. Nerve impulses from eighth nerve in response to single clicks. *C*. Response to single clicks as recorded from round window, consisting of cochlear response followed by nerve-impulse complex. *D*. The same, except for increase in strength of stimulus, after death of animal showing persistence of cochlear response and loss of nerve impulses. No response is obtained from the nerve after death.[18]

complex waves of the human voice are reproduced by it with the accuracy of a microphone, while from most nervous structures there is so much distortion and suppression of high frequencies that speech may be quite incomprehensible.

(2) The cochlear response is much more resistent to adverse circumstances, such as cold, anesthesia, lack of blood supply, and even the death of the animal, than are the nerve impulses. It was by virtue of these differences that the distinction between the two types of response was first established. Fig. 2 illustrates the persistence of the cochlear response with absence of the neural component after death of the animal. It also illustrates the separation in time of the two phenomena.

(3) The cochlear response appears without measurable delay after the arrival of a sound wave at the inner ear. The latency is certainly less than a tenth of a millisecond from the arrival of a sharp-fronted wave train at the tympanum, and we must still allow for the transmission of the waves through the ossicles and endolymph to the sense organ itself. The nerve impulses appear only after a further delay of approximately 0.7 millisec. These measurements are most conveniently made if we employ a sharp click as the stimulating sound. The click is generated by sending a condenser discharge through the stimulating loudspeaker which gives rise to a brief, highly damped wave train of high frequency, as revealed by the condenser microphone which samples the sound waves at a point closely corresponding to the position of the animal's ear. The cochlear response in this case does not reproduce the wave form of the stimulus, but consists of one large wave followed by from one to four damped oscillations at a frequency which seems to be more characteristic of the ear than of the stimulating wave train and considerably outlasts the stimulus. If the active recording electrode is placed on the round window, we see the first large wave of the cochlear response followed after about 0.7 millisec. by a second complex which is due to the nerve impulses. As the figure illustrates, the second complex vanishes after death, but not the first.

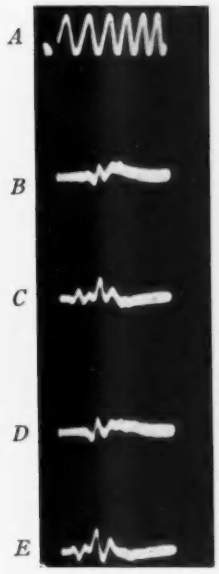

FIG. 3. *A*. 1000 c.p.s. wave. *B*. Action potentials from eighth nerve, representing nerve impulses in response to single clicks. *C*. Response from round window. Stimulus identical with that for *B*. *D*. Same electrodes and strength of stimulus as *B*, but polarity of stimulating sound waves reversed. *E*. Conditions identical with *C* except for reversal of polarity. Note similarity between *B* and *D*, and similarity of second components in *C* and *E* with reversal of first component in *E* with respect to *C*.[18]

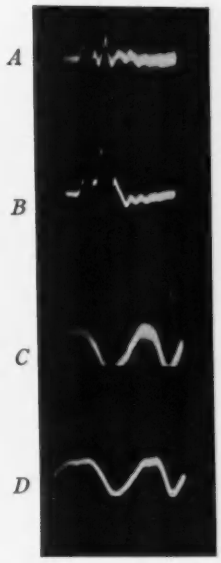

FIG. 4. Oscillograms from round window, showing masking. *B*. Response to clicks without masking. *A*. Same, masked with "hissing" sound. Note reduction of second component and constancy of first component. *D* and *C*. Same for continuous tone of 438 c.p.s.[18]

(4) Fig. 3 illustrates another point of difference. If the polarity of the stimulating wave train is reversed, the polarity of the cochlear response is also reversed, whereas the nervous component retains its characteristic polarity and form.

(5) If we deliver two sounds simultaneously to the animal's ear, such as a series of clicks plus a hissing sound, we see that the cochlear response and nerve response behave differently. In the cochlear response, the two stimulating patterns may be recognized summing with one another as they would appear if the microphone were employed for their detection instead of the cat's ear. The nerve impulses, however, suffer masking.

IV. MASKING[15, 18]

Fig. 4 illustrates the reduction of the nervous component by such masking of clicks by a hiss. Fig. 5 illustrates the interplay between clicks and a tone of 500 c.p.s. The situation is simplest and clearest in the record taken from the eighth nerve, consisting of nerve impulses alone. In this case both clicks and tone were delivered at an intensity of 20 db above threshold. It is evident

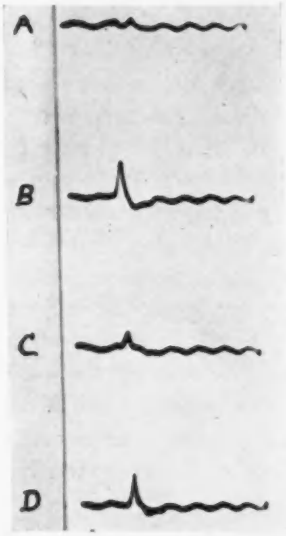

FIG. 5. Mutual interference of responses to single clicks and to continuous tone of 500 c.p.s., recorded from eighth nerve. *A*. First component of response to click almost completely masked, second component unmasked. *B*. First component of click unmasked. One cycle of response to tone masked. Second component of click much reduced. Note difference of phase relation between tone and click. *C* and *D*. Intermediate phase relations with intermediate degrees of masking.[18]

that the phase relation between the two is of primary importance in determining whether the

tone masks the click or the click masks the tone. The topmost tracing illustrates almost complete masking of the click, while in the next the click is as large as it appears when no tone is present, but one volley of the nerve impulses, which are synchronized with the tone, has been almost completely suppressed. The third tracing represents an intermediate phase relationship with partial suppression of each response. We have dubbed this relationship the "line-busy" effect.[21] Whichever stimulus first succeeds in setting up impulses in the fibers has right of way, for the second stimulus finds these fibers refractory and therefore sets up no impulses. This, we believe, is the fundamental mechanism of masking. It also shows, in this particular case, that click and tone are competing for the same nerve fibers. Masking depends primarily upon such competition.

V. Stimulating Phase

Careful study of such records as those shown here yields information on another interesting point, namely, the phase of the wave of pure tone during which nerve impulses are initiated. If the wave is pure, only one volley of impulses, more or less closely synchronized, is set up per wave and this occurs during the swing from electrical negativity to electrical positivity at the round window. By means of a condenser microphone sampling the stimulating sound wave, we have been able to correlate this phase of the electrical wave with the development of *negative* pressure in the auditory canal. In other words, nerve impulses are initiated during the outward but not during the inward movement of the ear drum. This is contrary to the tacit assumption which, I believe, most of us have previously made.

VI. Polarity of Basilar Membrane

Before leaving the cochlear response, we may note that its polarity with respect to an indifferent electrode placed elsewhere upon the animal varies with the position of the active electrode. By drilling through the bone so as to make contact directly with either the scala vestibuli or the scala tympani, we have found that the responses from these two channels near the round window are 180° out of phase with one another. When the scala vestibuli is negative, the scala tympani is positive, and *vice versa*. In addition to this, we[18] have preliminary observations which indicate a lag at the apex as compared with the round window, which, we believe, will be of great significance in determining the exact mode of vibration of the basilar membrane.

VII. Equilibration and Alternation[17, 18]

Returning to the auditory nerve and the question of the limit of frequency of impulses which it transmits, Fig. 6 shows the series of nerve impulses set up at the onset of a tone of 435 c.p.s. After a very brief initial transient, which we may call the "on-effect," the impulses take up the frequency of the stimulating wave. At first, there is a rapid diminution in the size of the responses, but this settles asymptotically to a steady state. If we examine the size of the initial wave after the "on-effect," and also the degree of shrinkage which occurs, as a function of frequency, we obtain the results illustrated in Fig. 7. The

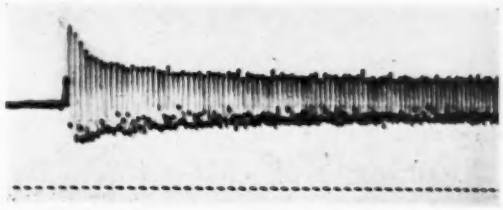

Fig. 6. Action potentials from eighth nerve in response to beginning of 435 c.p.s. tone. (Slightly retouched.) Note rapid initial shrinkage to a nearly constant size. Time marker equals 5 milliseconds.[18]

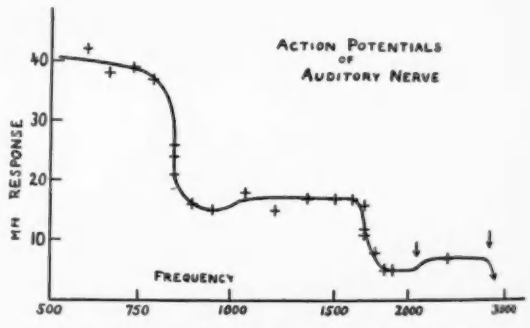

Fig. 7. Initial size of action potential in auditory nerve, measured immediately after the "on-effect," as a function of frequency (see text). The final plateau level is less than $\frac{1}{3}$ of the response at low frequencies. This is probably due in part to the fact that many of the nerve fibers respond quite irregularly and asynchronously at these frequencies, and therefore do not contribute to the size of the synchronized wave as measured.[18]

strength of stimulus is maximal throughout; that is, further increase in intensity of the stimulus causes no further increase in the size of the response except perhaps as it may introduce harmonics and consequent distortions of wave form. We call attention particularly to the sudden step-like drops with increase of frequency at approximately 800 and 1600 c.p.s. in this particular experiment. The higher steps regularly occur at frequencies which are 2 and 3 times that of the first step and the heights of the plateaus are usually in the approximate ratio of 1 to $\frac{1}{2}$ to $\frac{1}{3}$. Precise measurements are difficult to make on account of the rapid change which takes place in the size of the electric response. We may add that the rate of shrinkage and also the degree of shrinkage vary systematically with the frequency. The shrinkage, or "equilibration" as it is termed in physiological language, is greatest for frequencies just below those at which the fall in the size of initial response appears.

We believe that the interpretation of these relationships is as follows: Up to the frequency of the first drop, each nerve fiber can follow the frequency of the stimulating sound. However, during continued stimulation of the fibers near their limiting frequency their refractory period becomes longer. This is a well-known physiological fact for other nerves. Therefore the fibers cannot continue to respond at this rate and one after another they begin to respond only to *alternate* sound waves instead of to every one. This alternation accounts for the shrinkage in the size of the total response. If the stimulating frequency is somewhat higher, say 1200 c.p.s., none of the fibers can follow it in a 1-to-1 relation, but all of them respond to alternate waves. The composite wave is, therefore, about half what it is for a frequency of 600, but may be relatively well maintained as stimulation continues. At a still higher frequency, the fibers are forced to shift from a 1-to-2 to a 1-to-3 relation to the sound waves.

The situation is illustrated diagrammatically in Fig. 8, in which is pictured the hypothetical response of four typical nerve fibers, the vertical bars representing the size of the impulses, and the horizontal bars, the duration of the refractory periods, while the large vertical bars indicate the size of the composite response. The exact frequency at which the fibers go into alternation varies considerably with the temperature and general condition of the nerve, but in good fresh preparations the steps appear at approximately 1000, 2000 and 3000 c.p.s. We conclude that 1000 per second is the limiting frequency of impulses in the fibers of the auditory nerve, just as it is for other mammalian nerve fibers of similar anatomical characteristics.

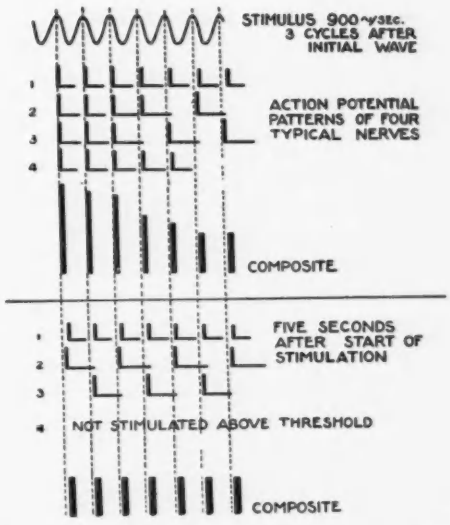

FIG. 8. Diagram illustrating responses of four typical nerve fibers to stimulation by a tone of 900 c.p.s. The size of the individual nerve impulses is represented by the vertical lines. These are smaller, because of greater equilibration, in the fibers which respond to every stimulus than in those which respond to alternate stimuli. The duration of the refractory period following each impulse is represented by the horizontal bar in each case. This prolongs with repetitive stimulation and in some fibers overlaps the next stimulus. Under these circumstances the fiber fails to respond until the next cycle. Fiber No. 4 is supposed to be at the margin of the excited area and to fail to respond because of the rise in threshold, which is regularly associated with stimulation at high frequency.[18]

VIII. THE BASIS OF LOUDNESS[11, 15]

It is interesting to note that there are apparently no subjective phenomena correlated with these critical frequencies. We might expect a greater susceptibility to fatigue corresponding to the equilibration, or an abrupt change in loudness with shift of frequency near the critical points. These have not been reported in studies of fatigue or loudness, and we must conclude that our judgment of loudness depends not upon the number of impulses per second arriving at the auditory nerve centers but instead upon the

number of fibers which are delivering impulses, irrespective of their frequency. This is contrary to the usual assumption based on analogy with other sense organs.

IX. Cortical Potentials

Another argument against a "telephone" theory of hearing is the failure to detect the corresponding audiofrequencies from the cortex of the brain. (The electrical activity of the cortex is a long story in itself.[2, 4, 7]) In one of our own experiments when the stimulating frequency was 20 c.p.s., there was a suggestion in the record taken from the auditory area of a series of waves at this frequency mixed with a great deal of activity difficult to characterize as to frequency or wave form. It is worth noting, however, that this tone to the human ear was definitely a flutter rather than a musical tone. Since we have not been able to detect cortical waves except for such very low tones of a fluttering or buzzing quality, the suggestion is obvious that the presence of discrete cortical waves is correlated with the sensation of discontinuity as opposed to the smooth quality of a musical tone.

X. Relation of Size of Response to Intensity

If we examine the magnitude of the cochlear response as a function of intensity of stimulation, we regularly find the type of relation illustrated in Fig. 9.[16, 18] The function is not simple. It is approximately linear at very low intensities, but for the entire range is more nearly hyperbolic. If size of response is plotted against the logarithm of intensity as in the figure, the curve is sigmoid, reaching a plateau at some 60 or 70 db above threshold. In the nerve it is difficult to make satisfactory measurements on account of the complications of alternation and equilibration which we have already described, but by measuring the initial response at suitable frequencies we find a similar relationship. This is also illustrated in Fig. 9. The amplification was adjusted to make the maximal response coincide with the maximal cochlear response. Apparently the number of nerve impulses set up is approximately proportional to the magnitude of the cochlear response. This conclusion follows from the fact that the

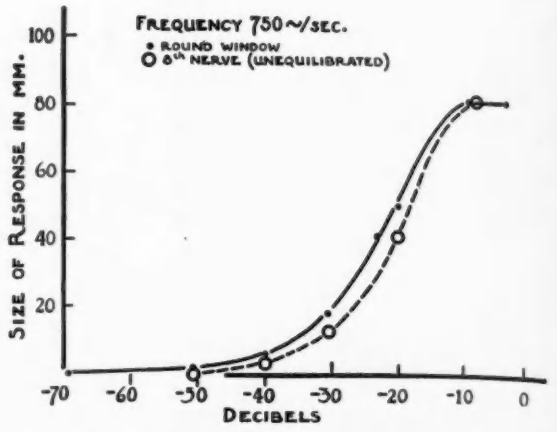

FIG. 9. Size of response as recorded from round window (dots) and from eighth nerve (circles) before equilibration, as function of intensity of stimulation. The zero of the intensity scale is the intensity of sound delivered to the animal when 1 volt (r.m.s.) is delivered to the loudspeaker. Responses are measured in millimeters on cathode-ray oscillograph and are proportional to voltages at the recording electrodes. The degree of amplification was adjusted to make maximal responses of nerve and cochlea coincide.[18]

nerve impulse is a definite quantum, all-or-none in character, incapable of gradation in size by alteration in the strength of the stimulus. Increase in the size of the response, therefore, implies addition of more fibers to the group already active.

Increase in intensity of stimulation beyond that necessary to attain the plateau will often introduce numerous harmonics, particularly in the case of low tones. The harmonics are introduced by the middle and the inner ear, and they greatly complicate the wave form with or without increasing the peak voltage of the response. Sufficient increase in intensity of stimulation causes an actual shrinkage of the response. This effect is only slowly reversible and seems to represent a fatigue of the auditory mechanism[32] or an actual overload with temporary damage. The depression is not specific for the stimulating tone but involves a very considerable range of frequencies both above and below. Long continued stimulation at high intensities leads to definite degeneration of sensory cells in the organ of Corti.[13, 14]

XI. Threshold Measurements

We have made a fairly extensive study of the threshold of the electrical response as a function

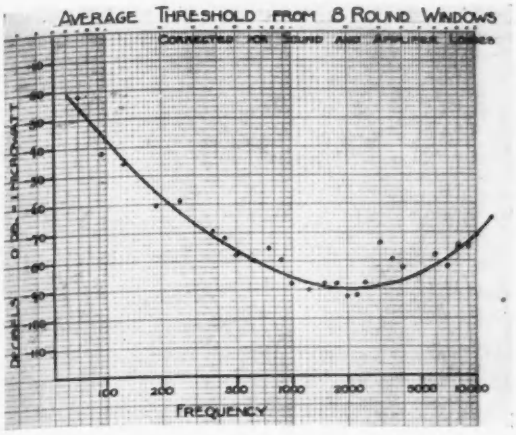

FIG. 10. Average audibility curve for 8 cats based on the threshold response detected from the round window. Corrected for frequency characteristics of sound system and of amplifier. Zero corresponds to 1 volt (r.m.s.) delivered to loudspeaker.[18]

of frequency under various conditions.[33] This type of measurement corresponds to the familiar curve of human audibility. As a matter of fact, the relationship turns out to be closely similar, both qualitatively and quantitatively. The average "audibility" curve from eight cats is shown in Fig. 10. The intensity scale is arbitrary, 0 db representing 1 volt delivered to the loudspeaker. We have compared the sensitivity of cats with human auditory acuity by putting the speculum used for the cats into our own ears and taking our own audiograms. The cats do very nearly as well as we do for the higher frequencies but not quite so well in the lower range (Fig. 11). We must remember, however, that we are recording from the round window and that the apical region of the basilar membrane is farther away from the recording electrode than is the basilar region. If low tones received are at the apex, this may account for the apparent discrepancy. We shall return to this point in a moment.

Similar observations on guinea pigs by Dr. S. S. Stevens and the writer are given in Fig. 12. The lower curve (dots) represents the crude data, while the upper curve (crosses) represents the divergence from normal human threshold. The guinea pigs seem to be as good as human beings for low tones but not as good for high tones. For comparison, the figure also shows (circles) the determinations of auditory acuity made by Horton[26a] at Princeton, using the method of

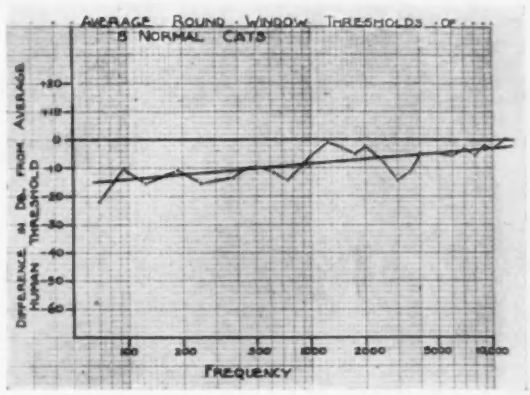

FIG. 11. Same data as in Fig. 10, expressed as "hearing loss" with respect to the average of normal human hearing as determined on the same stimulating apparatus.[18]

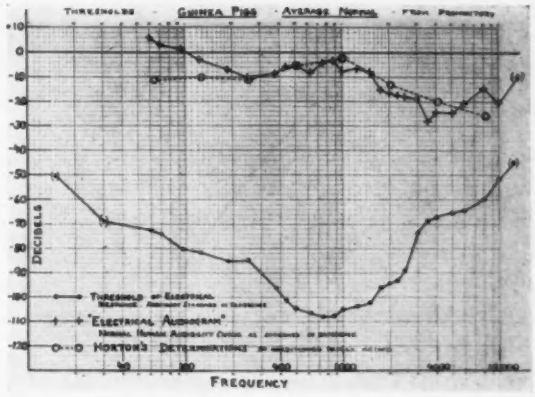

FIG. 12. Lower curve (dots), average audibility curve of 14 guinea pigs as determined by threshold of electrical response from round window. The lower the point, the greater the sensitivity. No correction for frequency characteristics of sound system, but corrected for amplifier losses. Upper curve (crosses), same data referred to average of normal human hearing determined on the same apparatus. This is comparable to the curve of Fig. 11 for cats. The lower the point, the less the sensitivity. Upper dotted curve (circles), average of Horton's determinations of auditory acuity of guinea pigs by the method of conditioned reflexes, also referred to average normal human hearing.[14]

conditioned reflexes. This method really shows what the animal hears and not simply what the sense organ is doing.

XII. Evidence for Specific Resonance

Fig. 13 shows average curves from guinea pigs, the solid curve representing data from the round window, whiie the dotted line represents the apex. In the guinea pig, the cochlea

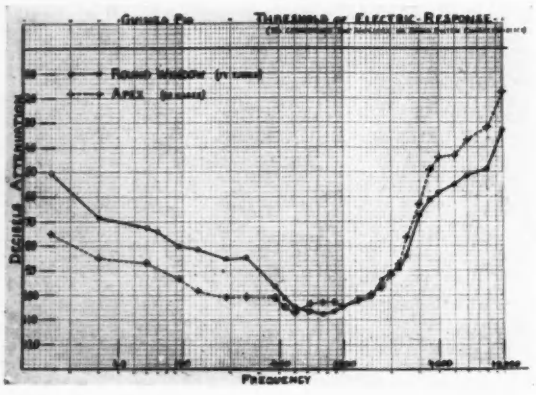

FIG. 13. Comparison of thresholds of electrical response from round window and from apex of cochlea in guinea pigs. No corrections. Note that the sensitivity is greater for high tones at the round window than at the apex, and greater for low tones at the apex than at the round window.[36]

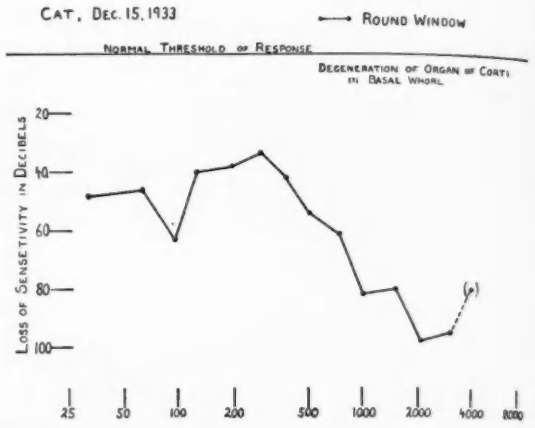

FIG. 14. "Electrical audiogram" ("hearing loss" in decibels) of a cat which, on microscopical examination, showed complete degeneration of the organ of Corti in the basal whorl but a fairly normal organ of Corti in the apical region. The zero of the sensitivity scale is the average threshold of response for normal cats.[33]

is exposed in the inner ear in such a way as to make the apical region readily available for experimentation. It is evident that there is a much lower threshold of response to low tones at the apex and to high tones at the round window. This confirms the previous observations on the cat by Hallpike and Rawdon-Smith[23, 24] who showed a similar differential in regard to the size of response to high and low tones. These data show conclusively that the cochlea is tuned in some fashion from end to end, the low tones being represented toward the apex and the high tones toward the base.

Additional evidence supporting a "place" theory of hearing is obtained from a study of audiograms of abnormal animals. In the first place, albino cats which are congenitally deaf show no electrical activity from the ear whatever.[27, 16, 28] On anatomical examination under the microscope, such animals are found to have no organ of Corti, or at best it is highly abnormal in character. One animal, fortunately for us,[33] showed a degeneration of the organ of Corti in the basal whorl but a fairly normal sense organ in the apical turn. Near the apex the nuclei of the sensory cells were clearly discernible, although the cells themselves were not completely normal. Examination of the basal turn revealed merely the basilar membrane and Reissner's membrane, but complete absence of the organ of Corti—tunnel, supporting cells, sensory cells and all. Its audiogram is given in Fig. 14, showing moderate loss of sensitivity for tones below 250 and then a progressive loss of high frequencies which becomes absolute above 3500.

Attempts have been made to correlate the apical region with the reception of low tones. The apical region has been damaged surgically by drilling or fracturing the bone or by the local application of heat. The effect on the electrical audiogram is as a rule surprisingly slight, and until better agreement as to the facts and better understanding of the principles are reached it is wiser to avoid any precise statement in advance of detailed reports.

In regard to the middle region of the organ of Corti, the situation is more satisfactory. We[13, 14] have been able to damage this region selectively by exposing guinea pigs for several weeks to a tone of 2500 c.p.s. at about 100 db above human threshold. Fig. 15 shows the electrical audiograms of three such animals. All show losses of sensitivity which are greatest in the band from 700 to 1500 c.p.s. Curiously enough, the region of greatest loss does not coincide with the frequency of the exposure tone, but rather with the region of greatest sensitivity in the normal guinea pig. Histological examination of these ears showed degeneration of the external hair-cells in a fairly wide zone, centering in the middle of the second turn. In the worst case, occasional degenerate cells were found all the way to the apex. In the

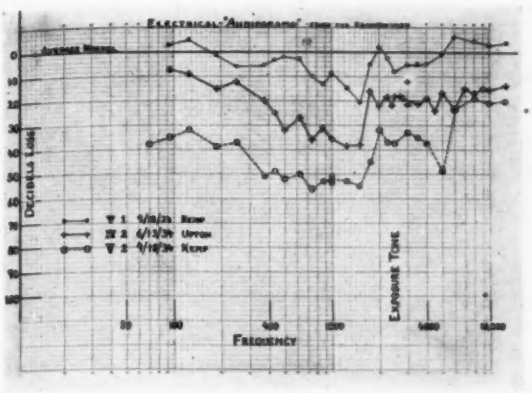

FIG. 15. "Electrical audiograms" of 3 guinea pigs which had been exposed to loud tones of 2400 or 2500 c.p.s. Upper curve (dots), exposed to 2400 c.p.s. at approximately 95 db for 15 hours per day for 40 days and to additional daily blasts of 3100 c.p.s. at approximately 85 db. Middle curve (crosses), exposed to 2500 c.p.s. at 106 db for 46 days. Lowest curve (circles), exposed to 2400 c.p.s. at approximately 95 db for 15 hours per day for 40 days and to additional blasts directly into the ear twice a day during the last 14 days. Each of these ears on microscopic examination showed degeneration of external hair-cells in the second whorl of the organ of Corti. The degeneration was most severe and most extensive in the animal corresponding to the lowest curve (circles) and least marked and most circumscribed for the animal corresponding to the upper curve (dots). Zero of reference is average threshold of response for normal guinea pigs.[14]

least severe case, the damage did not extend beyond the limits of the second turn. In no case was the internal group of hair-cells affected. This series correlates loss of sensitivity in the mid-region of the audible range with damage to the organ of Corti in the second turn, which is approximately half-way between round window and helicotrema. In some of our other cases more extensive or non-specific damage has been done to the inner ear by exposure of this sort, and widespread or complex losses of sensitivity ensued. This corresponds more closely with the results already reported by Wever, Bray and Horton[45] and by Finch and Culler.[18a]

XIII. Origin of Cochlear Response

The association of loss of sensitivity by our electrical method with degeneration of the sensory cells is further evidence that the cochlear response is generated by these particular cells.

The most plausible alternatives seem to be those advanced by Hallpike and Rawdon-Smith.[24, 25, 6] One alternative, ascribing the electrical effects to the vibration of a polarized membrane such as Reissner's or the tectorial membrane, seems to be eliminated by the absence of the response in albinotic cats and in their own recent case of a cat in which the eighth nerve had been severed with conservation of the blood supply to the cochlea and the neural elements allowed to degenerate. This animal yielded no cochlear response, but all structures except the nerve fibers and cells appeared normal under the microscope. On the basis of this case, Hallpike and Rawdon-Smith tentatively ascribe the cochlear response to the terminal branches of the nerve fibers in the organ of Corti. We believe that this interpretation is likewise impossible in view of the evidence we have outlined above as to the fundamental differences (e.g., resistance to death, latency, polarity, limits of frequency, fatigue and equilibration effects, and masking) between the cochlear response and the neural response.

Their case differs from those reported by Guttman and Barrera,[22] who found the cochlear response present for a (shorter) time following section of the nerve. We tentatively ascribe its absence in Hallpike and Rawdon-Smith's case to the onset of degenerative changes in the denervated sensory cells, in spite of their normal morphological appearance, and we adhere with confidence to our view that the cochlear response is generated by the sensory cells of the organ of Corti.

XIV. Summary

The final picture which we draw of the mechanism of the inner ear is somewhat as follows: The mechanical movements of the tympanum are transmitted through the ossicles to the fluid of the scala vestibuli. The basilar membrane is set in motion and the region most vigorously agitated depends upon the frequency of the vibration. A low tone causes vibration of a considerable zone toward the apical end of the membrane, while a high tone causes activity in a probably more sharply localized region near the round window. The membrane is not a series of resonators sharply set apart like the strings of a piano but presents a gradation in a continuous structure. The "on-effect" and "off-effect" which we see electrically indicate that the vibrating

mechanisms of the ear are almost, but not quite, critically damped. This also would militate against sharp tuning.

The mechanical vibration causes the sensory cells of the organ of Corti to develop an electrical potential between their upper and lower ends. The energy for this electrical potential is derived from the sound waves. Whether this cochlear response is part of the mechanism for the excitation of nerve impulses[16] or whether it has no physiological significance is still uncertain. Nerve impulses are set up during the phase of movement of the basilar membrane from the scala tympani toward the scala vestibuli, i.e., outward movement of the tympanum. The nerve impulses are synchronized with the sound waves as far as physiological limitations allow, but this depends on the intermittent repetitive nature of the stimulus and has no significance in the recognition of pitch. Differentiation of pitch depends upon activity of one or another zone of the basilar membrane and consequent differences in the distribution of activity among the fibers of the auditory nerve. Loudness does not depend upon the number of impulses per second traveling up the auditory nerve, but simply on the number of active fibers, irrespective of the frequency of the impulses in those fibers. The only significant limitation of hearing which we can definitely ascribe to the mechanism of transmission of nerve impulses is masking. Stimuli compete for the nerve fibers and, due to the refractory period of the nerve, one successful stimulus excludes response to a second stimulus for a significant, though brief, space of time. No other limitation has yet appeared that seems to be dependent on the quantal, intermittent nature of nerve impulses.

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A Loudness Scale for Industrial Noise Measurements

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Both on technical grounds and from experience in industrial noise measurements, it is recognized that the decibel scale, used as a loudness scale, does not yield numerical values proportional to the loudness sensation. In many spheres of work this occasions great inconvenience and there is need for an agreed rule for converting the decibel observations of practical noise measurements into loudness values on a linear loudness scale. A survey has therefore been made of published data with a view to deducing the most probable law for the relation between stimulus and sensation for a 1000-cycle pure tone. Relations derived from mental estimates of loudness, from data on monaural-binaural listening and from data on the properties of tone combinations are discussed. The evidence indicates that not only does the nerve-impulse theory of audition provide an explanation of the faculty of making mental estimates of relative loudness and of the monaural-binaural listening phenomena, but that stimulus-sensation relations deduced by the two methods are in substantial quantitative agreement. On theoretical grounds it is not thought that tone combination data are admissible for this purpose and an important divergence between these and monaural-binaural data appears to exist. Consequently the suggested relation put forward is not felt to be the most probable law. For the final relation derived in the present paper from the available data, the number 100 is assigned to the loudness evoked by an intensity of 100 db above 0.0002 dyne per sq. cm reference pressure. For practical industrial noise measurements, the relation is expressed sufficiently nearly by a simple power law between the limits of 30 and 115 db. Over this range, the loudness (L) is given by $L=d^5\times10^{-8}$, where d is the number of decibels above reference pressure. In addition to its being more in conformity with introspective judgments of relative loudness, this simple rule is thought to be preferable to the suggested curve because it is easily remembered, accurately definable and easily used with the aid of a slide rule.

I. INTRODUCTION

LOUDNESS is defined in the Report of the Committee on Acoustical Standardization of the Acoustical Society of America in the following way.

Loudness is that subjective quality of a sound which determines the magnitude of the auditory sensation produced by that sound.

The proposed British definition differs little from this and runs:

Loudness. That subjective quality of a sound which is associated with the strength of the auditory sensation produced by that sound.

The German tentative definition says simply:

Loudness. Strength of the sound perception.

All are agreed that loudness is a sensation rather than a physical quantity. Indeed, the sensation experienced must be the final criterion of loudness. All classes of the community are liable to annoyance from excessive noise and it is for the ultimate benefit of the general public that steps for the abatement or elimination of noise are taken. It is therefore necessary to give due consideration to the reactions of the intelligent non-technical person as well as to the acoustical technician in choosing a loudness scale for use in connection with noise problems. The issue that is continually before those who are concerned with industrial noise problems, especially those arising in engineering, is to what fraction of its existing loudness can a noise be reduced or, conversely, what steps are needed to reduce the loudness to a given fraction of its original value.

Long before the days of precise technique Weber's experiments suggested that, over a limited range of intensity at least, equal geometric increments of intensity give rise to equal arithmetic increments of sensation. Fechner integrated this relation and concluded that sensation magnitudes could be expressed by taking the logarithm of the intensity with reference to some datum point, the practically convenient datum in the case of sound being the threshold intensity. Transmission units called bels came into use in connection with electrical communication engineering and these, being logarithmic, were later adopted as sensation units. The more convenient decibel has been commonly used and loudness stated as so many decibels above a reference intensity approximating to the threshold intensity for persons of normal hearing.

In recent years the validity of the logarithmic intensity-loudness relation has come to be doubted. This arises from considerations of two kinds; experience in attempting to apply the logarithmic relation to practical problems of relative loudness and experimental evidence which has come to light since the relation was formulated.

II. The Decibel Loudness Scale

Let us consider the practical issues first. The engineer who attacks noise problems is frequently comparing one loudness magnitude with another and is in the habit of independently checking the conclusions of measurements. By introspection he makes rough judgments of the order of the relative loudnesses of different noises. If he is a person of normal hearing, he is perfectly justified in forming such judgments since the ultimate criterion of loudness can only be the magnitude of the sensation experienced by a representative listener. In common with others engaged on noise problems, the experience of the author and his colleagues over many years is that the numbers assigned by the decibel scale to represent sensation magnitudes are not acceptable to introspection as indicating their relative magnitudes. Two instances will suffice. The loudness of the noise of a motor assessed at 80 db above threshold in terms of a pure reference tone is, to introspection, enormously greater than twice that of a motor assessed at 40 db. In some experiments on the reduction of the noise of a geared turbo generator, successive measured values of 104 and 100 db were obtained or what would ordinarily be taken to indicate a 4 percent reduction in loudness. However, to the ear the loudness reduction appeared much greater than 4 percent and an onlooker who knew nothing of the measurements volunteered the opinion that the reduction was about 20 percent. On other evidence we know that it was actually about 27 percent.

From the published accounts of investigations carried out in U.S.A., it is evident that similar difficulties have been encountered. The tendency of opinion is always that the rate of increase of loudness with decibels above threshold is comparatively small at low intensities and very much greater at high intensities, with a result that the decibel scale appears very open at low and very cramped at high intensities. Beyond all question, judgments of relative loudness are frequently formed and have to be reckoned with and the continual discordance between the decibel loudness values and introspective loudness judgments is no small hindrance to progress in the solution of engineering noise problems.

In recording noise observations in decibels, we are only stating the intensity of the reference tone which evokes in a representative listener a loudness sensation equal to that of the noise under observation. This is, or should be, the case whether an objective or subjective method of measurement is used. While there is much to be said for continuing to make observations in this form, there is a pressing need for the relation between stimulus and sensation for the reference tone to be defined, so that the decibel stimulus observations may be interpreted and converted into loudness values, the values being numerically proportional to the sensation evoked in a representative listener.

Of the great value of the logarithmic or decibel scale for many purposes there is no question. It is a mathematical convenience, as it enables the large range of magnitudes arising in acoustics to be easily covered. It is useful instrumentally, as it enables direct reading attenuators to be readily constructed. Then in such questions as computing the effect of the distance of an observer from a pure tone point source, the convenient rule that the stimulus intensity falls by six decibels when the distance is doubled is available. A simple rule applies also in certain cases to the resulting intensity of a number of similar sources operating simultaneously. Again in considering the attenuation through a wall or partition, the intensity of a pure tone of given frequency will be attenuated by a fixed number of decibels independent, within wide limits, of the initial intensity of the tone.

In the cases quoted, the decibel scale enables the purely physical conditions to be dealt with expeditiously and it is not with this aspect of the matter that we are concerned here. We are concerned solely with the use of the decibel scale as a loudness scale.

It is now generally conceded by acoustical

technicians that the decibel scale of loudness or equivalent loudness is not a linear scale of sensation. Its continued use is now defended mainly on grounds of expediency. Since we have already considered the practical objections to this scale, we need not therefore dwell at any length on objections arising out of the known characteristics of the ear. Briefly, these are as follows:

The use of the scale to indicate loudness magnitudes, with its implication that the relation between decibels and loudness is linear, is based on the assumption that the minimum perceptible changes of intensity at different intensity levels are equal increments of sensation. With both intensity change and intensity level expressed in logarithmic units, the minimum perceptible change would be the same number of decibels at all levels if this were true. Experimental evidence,[1–5] shows that the minimum perceptible change varies from nearly 3 db at low intensities to a fraction of a db at high intensities. Even though the minimum perceptible intensity changes are known for different levels, the relative magnitudes of the sensations corresponding to them are not known. Further there is nothing to support the assumption that increments of sensation are additive.

It has also been pointed out[6] that if the Weber-Fechner law were true, the equal loudness relations for different frequencies would be represented by a family of straight lines converging to zero. This is not the case, as is well known from Kingsbury's work,[7] which has been substantially confirmed by other investigators. These considerations show that the decibel scale has no foundation as a loudness scale.

III. The Relation Between Stimulus and Sensation

The case which is of most interest is that for a pure tone in free space of 1000 cycles per sec. listened to with both ears, the listener facing the source. Since the numbers assigned to sensations are purely arbitrary, we can use any manageable and easily remembered range of numbers to express loudness magnitudes. We shall therefore call the loudness evoked by the reference pressure zero and that evoked by a stimulus of 100 db above the reference pressure 100. The reference pressure has been taken as 0.00020 dyne per sq. cm, which does not differ materially from that corresponding to the proposed U.S.A. standard reference intensity of 10^{-16} watt per sq. cm, *viz.*, 0.000207. Also it has been shown[4, 8] that the average free space threshold pressure for 1000 cycles for persons of normal hearing is substantially in agreement with this figure.

The principal methods available for determining the stimulus-sensation relation are those involving mental estimates of relative loudness and those based on a comparison of monaural and binaural listening. The evidence available from these methods is considered below.

(a) Deductions from mental estimates of relative loudness

Although this method may not be capable of a high degree of precision, it is of fundamental importance, since it makes a direct appeal to the final criterion of relative loudness, *viz.*, the individual's assessment of the sensations he experiences. The method seeks to obtain a representative value for the intuitive judgments of relative loudness of a group of individuals. The existence of the faculty of making quantitative estimates of relative loudness by intuition frequently occasions no little surprise. This is especially the case with persons who, initially doubting their ability to form such estimates,

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find they are able to do so with a fair consistency. Dr. R. T. Beatty has suggested[9] that the faculty is associated with the number of nerve impulses per second produced by the stimulus. In the case of lifting weights the number of afferent nerve impulses per second is proportional to the weight lifted. In Dr. Beatty's words:

> A muscle is composed of individual fibres, each of which is either relaxed or completely contracted. If 10 fibres must contract when a 1 lb. weight is lifted, 20 are required for 2 lb. with a corresponding doubling of the nerve impulses per second. The sensations evoked are correlated with the knowledge that in one case one object is lifted, while in the other case two similar objects are lifted and so a numerical ratio is assigned to the magnitudes of the sensations, a ratio which is identical with the ratio of the frequencies of the nerve impulses.

Hence, if we assume that this interpretation of sensory messages is transferred more or less accurately to the domain of hearing, we have an explanation of the faculty of intuitively estimating relative loudness magnitudes.

The method of mental estimates has been employed by several investigators[10–13, 4] and we shall now compare the results they have obtained.

The work of Richardson and Ross appears to be the earliest published. Sounds of a given pitch were produced by means of a telephone receiver applied to the subject's ear. The intensities of the stimuli corresponding to the loudness magnitudes estimated by the observer are given in terms of the telephone receiver input current, so that the stimulus magnitudes are not definitely known in terms of acoustical quantities. The loudest sounds required about 10,000 times the current for almost inaudible sounds, which suggests an upper limit of the order of 80 db for the stimulus. The authors concluded that for frequencies between 550 and 1100 loudness varied as the 0.44 power of the current, that is, of the acoustical pressure, if we may assume the linearity of the receiver. Extreme values of 0.24 and 0.73 were obtained. Omitting frequencies below 700, a mean index of 0.468 was obtained with nine subjects. The authors take the view that the decibel is a special measure of stimulus and is definitely not a unit of sensation. The paper deals admirably with the fundamental subjective considerations involved and it is to be regretted that the physical measurements are not in a form enabling the results of the investigation to be compared with those of other workers. Nevertheless, if an upper stimulus figure of 80 db be assumed, the results of Richardson and Ross lie within the spread of those of other investigators, as will be shown later.

In the experiments of Laird, Taylor and Wille the sound was apparently presented to the subject by means of a single telephone earpiece. Results are given for pure tones of 4096 and 1024 cycles, based on the reduction in stimulus required to halve the loudness, in the estimation of the subject. Ratios other than halving are not employed with pure tones. With complex tones there is a large divergence between the stimulus-sensation relations deduced from halving and quartering. The stimulus-sensation relations deduced from the 4096 and 1024-cycle pure tone data are so much at variance with those derived from other sources that they have not been utilized here.

Ham and Parkinson employed tones set up by an electrodynamic loudspeaker. Experiments were carried out with sustained pure tones, warble tones and complex tones but only the former will be considered here. Subjects uninfluenced by familiarity with the decibel scale were employed. It is not clear how many subjects took part in a given experiment, but a total of 175 was employed in all the experiments. In the 1000- and 2500-cycle pure tone tests, over 50 readings were taken for each stimulus value. The procedure was to start with a certain stimulus value and after reducing it by a known number of decibels, to ask the subject to estimate the loudness remaining, expressed as a percentage of the initial loudness. Care was taken to ensure that the subjects had no knowledge of

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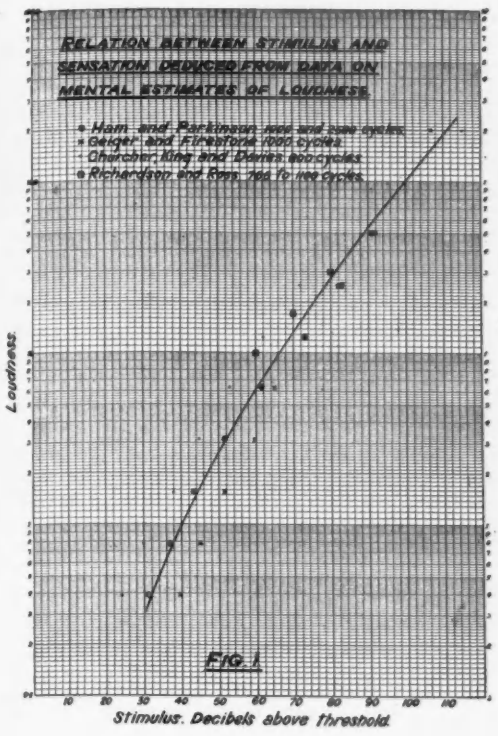
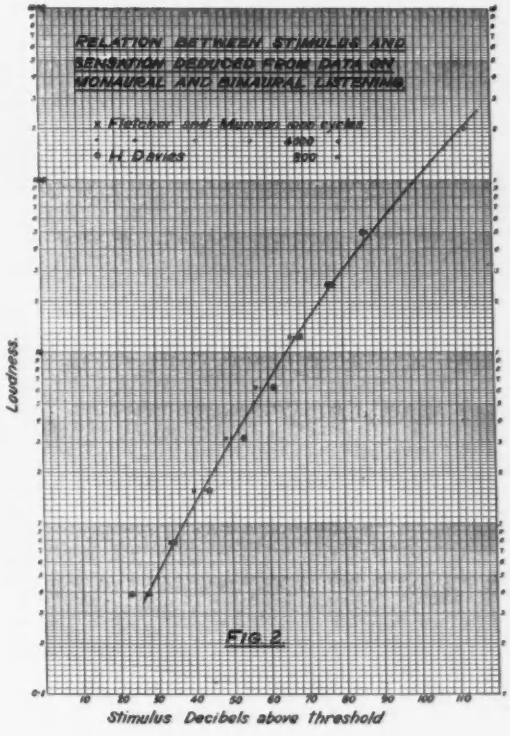
each other's results. The decibel values given are stated to be decibels above the threshold of hearing and it would appear that a threshold value of about 10 db below 0.001 dyne per sq. cm or 0.00031 dyne per sq. cm is indicated. As a given number of decibels above threshold give the same loudness at both 1000 and 2500 cycles, the results for these frequencies are directly comparable. The two sets of results were therefore plotted together on a basis of 0.0002 reference pressure at 1000 cycles, utilizing both "high level" and "low level" observations. Except at the lowest intensities the agreement between the 1000- and 2500-cycle results is good, the discrepancies in the intensity values being in most cases well within 4 db. A mean curve for the two frequencies has been drawn as representing Ham and Parkinson's pure tone results, the curve being extrapolated from 90 to 100 db in order to permit comparison with other results. From a consideration of the shape of these and other curves in this region, no material uncertainty is involved in the extrapolation. Points from this mean curve are shown in Fig. 1. The pure tone tests at 350 cycles have not been used, as it would have been necessary to introduce a correction to bring them to the 1000-cycle basis.

In the experiments of Geiger and Firestone the test tone was presented to the subject by means of telephone receivers applied to both ears. The subject varied the test tone but was not allowed to see his settings. He was asked to adjust for various loudness ratios of 1/2 to 1/100 and from 2 to 100. The results are given in terms of a threshold pressure of 0.00053 dyne per sq. cm for 1000 cycles. Results are given for 1000 cycles corresponding to initial intensities of 38, 63 and 88 db above 0.0002 dyne per sq. cm. Curves have been plotted together for these three sets of results, an intensity of 100 db above threshold being designated 100 loudness in each case. The agreement between the curves is fairly good, the maximum discrepancy being 8 db for a given loudness. The authors stress the fact that they found certain inconsistencies in comparing the different experimental procedures. For instance, if a subject is asked to adjust the intensity for double loudness and then asked to adjust for halving the latter, he will not, in general, return to the initial intensity but to a value higher or lower according to the order of intensity level under examination. There is no doubt that as the authors state, mental estimates are conditioned to some extent by the procedure adopted. However, the observations, when plotted, yield quite smooth curves. There is no evidence to show whether multiple or fractional estimates have the greater weight. Since each set of observations contains both types of estimate at different levels and covers nearly the same intensity range, the three sets have been considered as of equal importance and a mean curve plotted. It is found that none of the points from the three sets of observations deviates from this curve by more than 4 db. The points from this mean curve are shown in Fig. 1. The 1000-cycle results are stated to be based on 44 observations but it is not clear how many subjects were employed.

The experiments of Churcher, King and Davies were carried out mainly on 800-cycle pure tones, the test tone being presented by means of a pair of telephone receivers. The linearity of these receivers was checked up to 112 db. After some preliminary experiments to determine the consistency of the results on a single subject, a series of observations on 34 subjects who had no previous experience of acoustical measurements was carried out. Two attenuators and a throw-over switch were so arranged that the subject could switch over from an initial intensity value, which was set for him, to another value which he could adjust without seeing the settings. Starting at 100 db above threshold, the subject was asked to adjust the second attenuator until he judged the tone to have half the loudness of the initial value. The first attenuator was then adjusted to the setting arrived at on the second and the subject again asked to halve the loudness. Thus by a succession of halvings an intensity-loudness curve was obtained. A similar set of observations on 30 unskilled subjects was carried out, the loudness being reduced to one-quarter the initial value instead of one-half. The results obtained are summarized in Table I.

It is seen that there is no material difference between the results obtained by halving and those obtained by quartering. Some further observations were carried out to extend the results up to 110 db and down to 15 db and points from


RELATION BETWEEN STIMULUS AND SENSATION DEDUCED FROM DATA ON MENTAL ESTIMATES OF LOUDNESS
Loudness
Stimulus. Decibels above threshold
FIG. 1.


RELATION BETWEEN STIMULUS AND SENSATION DEDUCED FROM DATA ON MONAURAL AND BINAURAL LISTENING
Loudness
Stimulus Decibels above threshold
FIG. 2.

the final curve obtained are given in Fig. 1. Curves determined at 200 and 2000 cycles were found to give good agreement with those at 800 cycles when converted in accordance with Kingsbury's equal loudness relations.

In Fig. 1 three points deduced from the paper of Richardson and Ross are plotted on the assumption that the maximum intensity reached in those experiments was 80 db. It is seen that these points lie within the spread of the others but no deductions can be made from them because of the uncertainty in the intensity values.

TABLE I.

Loudness	Intensity by halving (db)	Intensity by quartering (db)
100	100	100
50	84	—
25	72	70
12.5	62	—
6.25	52	54
3.13	43.5	—
1.56	35.5	40

The three remaining sets of points, which have been derived from independent investigations, show a measure of agreement which is reasonable, considering the nature of the subject. Each investigation was carried out with a different technique, under different experimental conditions and with different subjects. It would seem that an exhaustive study of the influence of these factors would have to be undertaken before much closer agreement could be expected. The mean of the three sets has therefore been taken as representative of the stimulus-sensation relation as deduced from the mental estimate data at present available. The mean curve is shown in Fig. 1. At 100 loudness, the curves have been made to coincide. At 10 loudness, the maximum deviation between the mean curve and that representing any individual investigation does not exceed 7 db. At a loudness of 1 the deviation is still within 7 db, increasing to about 8 db at 0.4 loudness. It may be noted that Laird, Taylor and Wille's data yield a curve which at 10 loudness deviates from the above mean curve by approximately 30 db.

(b) Deductions from monaural and binaural listening data

In a recent paper,[14] Fletcher and Munson give data based on a comparison of monaural and binaural listening. The stimuli that produce a given loudness when applied to one and to both ears, respectively, are deducible from Fig. 7 of their paper. In accordance with the nerve impulse theory the hypothesis is put forward that when a given stimulus is applied to both ears it produces twice the number of nerve impulses per second, and hence twice the loudness sensation, as when the same stimulus is applied to one ear. On the basis of this hypothesis a stimulus-sensation relation may be derived. Particulars are not given as to precisely how the data of Fig. 7 are obtained. Points on the graph are shown for 125, 1000 and 4000 cycles. By using the 1000-cycle points, a curve has been derived, the points of which are shown in Fig. 2. As 1000- and 4000-cycle data are directly comparable, a stimulus-sensation curve has been similarly deduced from Fletcher and Munson's 4000-cycle points. Points from the 4000-cycle stimulus-sensation curve are also shown in Fig. 2. It is seen that the 1000- and 4000-cycle points of Fig. 2 are in close agreement. The curve derived from Fletcher and Munson's 125-cycle points was also examined. This shows rough agreement with the curve determined directly from 1000-cycle data. However, for our present purpose, it is not necessary to use the 125-cycle data.

Reference has already been made to some experiments[4] carried out on the minimum perceptible change of intensity at different intensity levels. These experiments, which are discussed in detail in a later paper,[5] were carried out with a view to constructing a curve connecting stimulus and the number of just perceptible increments of intensity. This matter is discussed in the paper referred to and need not be enlarged upon here. It is sufficient to say that the curve in question was not accepted as defining a relation between stimulus and sensation. In a subsequent paper[6] it was suggested that the minimum perceptible change data which had been obtained for monaural and binaural listening might be utilized to derive a stimulus-sensation relation. In addition to the hypothesis that a given stimulus applied to both ears sounds twice as loud as when applied to one, it is necessary for this purpose to make a further assumption. The assumption is that the value of the minimum perceptible change of intensity depends upon the magnitude of the sensation and hence, as a corollary, that when different methods of applying stimuli produce the same value of minimum perceptible change, the sensation magnitudes produced by these methods of application are also equal. In the paper under reference 14, minimum perceptible change data are given for both discrete and cyclic changes of intensity. It was found that the cyclic changes were more definite and required far less effort on the part of the subjects than the discrete changes. The cyclic changes only, which are shown in Fig. 3 of that paper, will be used here. As might be expected, the minimum perceptible change of intensity at a given level is larger for monaural than for binaural listening. Observations were taken up to 90 db. For our purpose it is desirable to know the monaural minimum perceptible change at 100 db and to obtain this the curve has been extrapolated. A consideration of the shape of the curve and the range of the extrapolation shows that no material uncertainty is involved. The points on the stimulus-sensation curve deduced from the two minimum perceptible change curves are shown in Fig. 2. The good agreement with the other two sets of points derived from tests on the relative stimulus intensities for equal loudness with monaural and binaural listening is a strong argument for the validity of the assumption made in regard to the dependence of the minimum perceptible changes on the sensation magnitudes.

In making a final deduction from these data, it is not easy to decide what weight to give to the curve obtained from minimum perceptible changes compared with the others. Fletcher and Munson do not state the number of subjects employed in their experiments but the close agreement between the 1000- and 4000-cycle data is to be noted. Only four skilled subjects

[14] Harvey Fletcher and W. A. Munson, J. Acous. Soc. Am. **5**, 82 (1933).

were employed in obtaining the minimum perceptible changes and the spread of the results was appreciable. On the other hand, no results, however obtained, are free from experimental error. A weighted mean curve has therefore been drawn in Fig. 2, the results based on the minimum perceptible changes being given half the weight attached to each of the other two sets of results. It is clear from Fig. 2 that whether we take this view of the matter or whether we leave the minimum perceptible changes out of account altogether, the conclusion would not be materially affected. We may therefore take the curve as representing the stimulus-sensation relation as deduced from monaural and binaural listening data at present available.

(c) Tone combination methods

Fletcher and Munson have suggested[14] that the relation between stimulus and sensation may be deduced from a consideration of the resulting loudness of combinations of tones. A curve has been put forward[15] which is largely based upon these considerations. The method involves the acceptance of a particular theory of that very complex subject, the response of the ear to combinations of tones. Until such theory has been demonstrated to be generally valid, the use of the method for deducing a fundamental relation for pure tones is hardly admissible. That there is doubt as to the general validity of the method is shown by the following considerations.

Take the case of a complex tone of n components, the intensities of the several components, when expressed in terms of the equivalent reference tone (1000-cycle) intensities (Lk), being equal. Then Fletcher and Munson take the view that the presence of one component diminishes, in some degree, the loudness contributions of the others, but that when the components are sufficiently widely separated in frequency, the nerve terminals stimulated by each component are at different portions of the basilar membrane and hence that the interference of the loudness of one tone upon that of the others is negligible. For negligible masking conditions, the following equation is given:

$$G(L)=nG(Lk),$$

where G is a function connecting intensity and loudness and L and Lk are the intensities of the reference tone and the component tones, respectively. Thus $G(Lk)$ is the loudness of each of the equally loud components and $G(L)$ is the loudness of the reference tone. Hence, under the conditions specified, the total loudness is n times the loudness of each component, from which it follows that if the total loudness of a known combination of tones is measured, the stimulus-sensation relation for an individual tone can be deduced from it.

However, there is a very important factor of which Fletcher and Munson's treatment takes no account. This is whether the frequencies of the component tones are harmonically related or not. The importance of harmonically related components was brought to the attention of the author and his colleagues during their study of the noise emitted by electrical apparatus. From the results of measurements on apparatus of different types, comparisons were made between the intensity of the 800-cycle pure reference tone which sounded as loud as the noise emitted and the total equivalent energy of the components when referred to a common 800-cycle basis and expressed in decibels. Except for the use of 1000 cycles instead of 800, the latter is the principle on which the weighted network objective noise meter operates. With certain types of machines, e.g., turbo-generator sets, reasonable agreement between subjective and objective data was obtained. The discrepancies did not exceed 10 db in 90 and were a good deal less in most cases. But with transformers, discrepancies up to 31 db were obtained, the subjective value always being considerably larger than the objective value. The distinctive feature of transformer noise is that there is present a purely harmonic series of tones. For a 50-cycle transformer the frequencies present are 100, 200, 300, 400, and so on. It was concluded that the large increase in the subjective value for a given objective value was due to the non-linearity of the ear, which gives rise to subjectively formed combination tones which are not objectively present. When harmonic components of the same frequencies as the com-

[15] *Proposed Standards for Noise Measurement*, J. Acous. Soc. Am. **5**, 109 (1933).

bination tones are objectively present in addition, the subjectively formed tones may be reinforced, giving a greatly increased loudness sensation, especially when the number of components is large. These results were published[16] and some subsequent laboratory experiments[4] with synthetic tones showed that these effects were not peculiar to transformers. For instance, in the case of a harmonic series of tones with a fundamental of 108 cycles the fundamental was so large that the energies of the harmonics were negligible and the total energy was indistinguishable from the energy of the fundamental alone, which was equivalent to 92 db. The subjective value of the series was 102 db. On removing the harmonics, the subjective value fell to 90 db, i.e., the removal of components which did not appreciably contribute to the total energy produced a change of 12 db in the subjective value. For a further discussion of this matter, the paper referred to should be consulted.

The conclusion is that it is not possible to deduce the total loudness of a number of components irrespective of whether their frequencies are harmonically related or not. This will be the case whether the summation be visualized on an energy or on a loudness basis. That harmonically related components may be expected to produce special effects is evident from the graphs given on p. 169 of Fletcher's *Speech and Hearing*. The proposed standard curve referred to is obtained by averaging the results of three methods, *viz.*, monaural-binaural, 2-tone and 10-tone combinations. The 2 tones were in harmonic relation and the 10 tones had a constant frequency separation. The assumption is made that, apart from experimental errors, the three methods should yield the same result. But the dependence of the loudness of a complex sound on whether the component frequencies are harmonically related or not and the absence of any allowance for this effect in Fletcher and Munson's treatment leads to the expectation that any agreement between monaural-binaural and tone combination results will be fortuitous. An examination of the experimental points given and a replotting on a loudness basis shows that while the 2-tone curve coincides approximately with the monaural-binaural curve, the 10-tone curve shows a wide discrepancy. The discrepancy may be due to harmonic relations or to the assumption that masking was negligible in that case or both. The discrepancies shown in Figs. 6 and 7 of Fletcher and Munson's paper do not appear serious until it is realized that an error of a few decibels in the ordinates can cause a very large error in loudness. For our present purpose we shall not therefore utilize tone combination data.

Fletcher and Munson's theory is a notable contribution towards the solution of a supremely difficult problem but, as the authors themselves indicate, modification in the light of further experience may be necessary.

IV. Comparison of Mental Estimates and Monaural-Binaural Results

We can now compare the conclusions from mental estimate data with those from monaural-binaural data. This has been done in Fig. 3, where the crosses represent points taken from the mean curve of Fig. 1 and the dots those from Fig. 2. It is seen that there is no serious divergence between the mental estimates and the other methods; in fact the agreement exceeds expectations. While in the author's view the stimulus-sensation relation derived from mental estimates is a fundamentally important guide, in that it makes a direct appeal to the final criterion of relative loudness, *viz.*, the representative individual's assessment of his sensations, yet it is clear that if a factor of exactly 2 is involved in the monaural-binaural comparison, the stimulus-sensation relation can be realized with greater precision from monaural-binaural data than from mental estimates, owing to the difficulty of closely defining the relevant technique for obtaining the latter. If it had been found that relations deduced by the two methods were in conflict, there would have been grounds for suspecting the applicability of the nerve-impulse hypothesis but since the agreement is good, the factor 2 may be accepted.*

[16] B. G. Churcher, A. J. King and H. Davies, Nature **132**, 350 (1933).

* The possibility of the factor being slightly less than 2 because of one ear responding when the stimulus is applied to the other has been considered but from data available this effect would appear to influence the results by only about 1 percent.

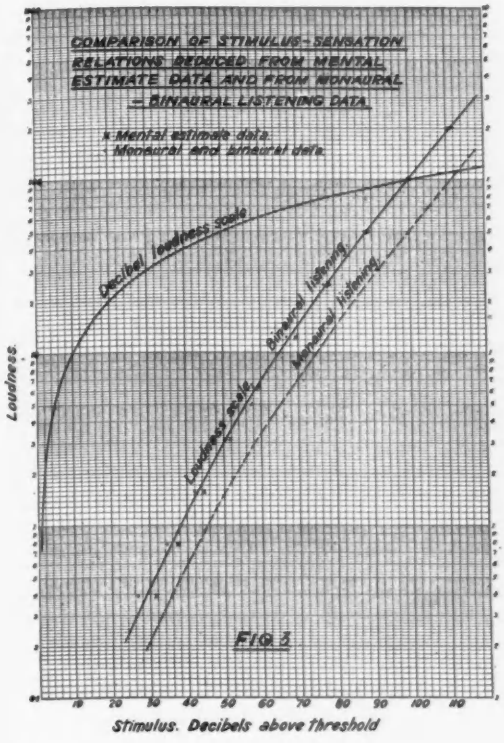

COMPARISON OF STIMULUS-SENSATION RELATIONS REDUCED FROM MENTAL ESTIMATE DATA AND FROM MONAURAL = BINAURAL LISTENING DATA
x Mental estimate data
· Monaural and binaural data
Decibel loudness scale
Binaural listening
Monaural listening
Loudness scale
Loudness
Stimulus. Decibels above threshold

FIG. 3

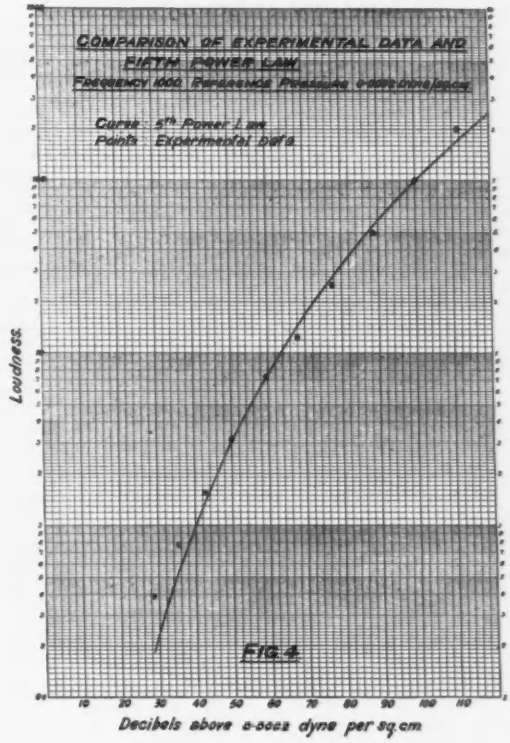

COMPARISON OF EXPERIMENTAL DATA AND FIFTH POWER LAW
Curve: 5th Power Law
Points: Experimental Data
Loudness
Decibels above 0.0002 dyne per sq cm

FIG. 4

In endeavoring to reach a final conclusion on the stimulus-sensation relation for 1000 cycles, the author considers that it would be unwise to ignore the mental estimates conclusions altogether in view of their being based on so large a mass of data and also because monaural-binaural conclusions are not exempt from experimental error. The full line curve shown in Fig. 3 has been arrived at by giving the monaural-binaural conclusions twice the weight assigned to the mental estimates conclusions. This curve, in the author's view, is the most probable relation between stimulus and sensation for a representative individual, listening with both ears, other conditions being a 1000-cycle pure tone and a reference pressure of 0.0002 dyne per sq. cm in free space. The number 100 is assigned to the loudness set up by a stimulus of 100 db above the reference pressure. The corresponding curve for monaural listening is shown by the dotted line. For the sake of comparison the decibel loudness scale is also plotted. The wisdom of attaching the word "loudness" to this scale may be judged by the reader.

It has frequently been contended that, although we can judge when two sensations, such as sounds, are equal or whether one is greater than another, to say that one sound is x times as loud as another has no meaning. In this connection it is to be noted that the nerve impulse theory provides an explanation of the faculty of making intuitive estimates of relative loudness and of the monaural-binaural listening phenomena and, further, that deductions from observations on these two phenomena show substantial quantitative agreement.

It has been rightly pointed out that sensations cannot be "measured" in the sense that physical quantities are measured and a great deal of discussion has taken place on this point. However, the question appears to be entirely a matter of terminology and should not be allowed to retard the development of the subject.

V. A Simple Rule for Interpreting Industrial Noise Measurements

For interpreting practical decibel observations, whether obtained by a subjective or an objective method, a simple and clear-cut rule is required. An equation is more precise and convenient than a curve. An examination of the final curve given

in Fig. 3 shows that it could only be completely represented by a rather complex expression. For practical purposes it is far more important to have a simple agreed rule than one of the maximum accuracy and a simple power law fits the curve well enough. In Fig. 4 the curve represents a fifth power law and the points are those from the final curve of Fig. 3.

It is seen that from 100 down to 1 loudness, the points fit the curve as closely as the accuracy of the experimental data allows and that even from 200 down to 0.3 loudness, no divergence that would be of importance in practice occurs. The latter range, which corresponds to about 115 to 30 in decibels, covers nearly all industrial and other practical requirements. For the interpretation of practical measurements, the following simple rule is therefore suggested.

$$\text{Loudness} = d^5 \times 10^{-8},$$

where d is the value of the intensity of the equivalent 1000-cycle pure tone in decibels above the reference pressure adopted, i.e., 0.0002 dyne per sq. cm. The rule is easily remembered and loudness values can be rapidly obtained with a slide rule or even by mental calculation for rough purposes.

We have already noted in Fig. 3 the comparison of the decibel loudness scale with that derived from modern experimental evidence. The db scale is not a loudness scale in any ordinary sense but simply a logarithmic scale of stimulus. It certainly arranges loudnesses in the order of their magnitudes but does not enable relative loudnesses to be indicated except by the somewhat unscientific expedient of calibrating the scale in terms of well-known noises. For rough purposes the decibel loudness scale presents no difficulty to those who are able to become familiar with it. The acoustical technician who becomes intimately acquainted with it, can interpret it mentally for his own purposes, but it is exceedingly improbable that the great majority of persons, who are non-technical from an acoustical point of view and who may have to consider the results of noise measurements, have either the inclination or the opportunity of acquiring such a faculty. In the author's opinion the attaching of the word "loudness," denoting a subjective entity, to the decibel scale, which expresses a purely physical quantity, is to be deprecated both for the reasons already given and because it tends to confuse cause and effect, i.e., stimulus and sensation. It should be our aim to assist in the development of acoustical science by maintaining clear distinctions.

We can continue to give the results of noise measurements and permissible noise limits in terms of the stimulus value of the equally loud reference tone, i.e., in decibels above reference pressure but we shall avoid a misnomer by not using the word "loudness" in connection with this quantity. A loudness scale such as that suggested will enable the loudness to be immediately obtained from a noise measurement or, conversely, the decibels corresponding to a given permissible loudness. The quantity we speak of will then be in conformity with the definition of loudness and we shall avoid the anomaly of loudness being defined as a sensation and expressed as a physical quantity approximately proportional to the fifth root of the loudness. We shall also promote the practical solution of noise problems by being able to express the results of measurements in a form acceptable to the layman.

Subharmonics in Forced Oscillations in Dissipative Systems. Part I

P. O. PEDERSEN,* *Copenhagen*

(Received September 5, 1934)

1. THE OBSERVATION OF THE PHENOMENA OF SUBHARMONICS[1]

THE phenomena of subharmonics were encountered for the first time at the Laboratory of Telegraphy and Telephony of the Royal Technical College, Copenhagen, during the testing of a moving-coil loudspeaker which was to be used for certain acoustic experiments. The loudspeaker was driven by a heterodyne generator the voltage of which was very nearly sinusoidal (with less than 2 percent harmonics) and which had a frequency range continuously variable from 0 to 10,000 cycles per second.

At the maximum voltage (i.e., the maximum output of the generator or approx. 5 watts) it was noticed that at certain frequencies the loudspeaker (a "Jensen Auditorium Speaker") in addition to the note corresponding to the frequency of the generator and some higher harmonics also produced another vibration of a higher or lower tone with a frequency one-half that of the former—in other words a 2nd subharmonic. The frequencies in question were as follows:

1800	1650	1520	1330	1150	825	650	490
or 11·164	10·165	9·169	8·166	7·164	5·165	4·163	3·163

It will be noticed, that the frequencies are very evenly distributed.

The subharmonic is not produced until the voltage reaches a certain threshold value and its intensity then appears to be almost independent of the value of the voltage. The subharmonic does not rise immediately to its full intensity; if the voltage only slightly exceeds the threshold value it may even be a matter of a few seconds before the subharmonic has reached its final amplitude.

A few figures would perhaps more easily illustrate the phenomenon. Fig. 1 is a schematic representation showing the general occurrence of the subharmonics. The horizontally shaded parts show corresponding values of current and frequency at which the 2nd subharmonic is produced. If the loudspeaker current gets below I_0 no frequency will produce subharmonics. If the current exceeds I_0, e.g., if it reaches I_2, the subharmonics will appear, not only for one or more single frequencies, but for one or more bands of frequencies.

At great currents some of these bands of frequencies merge and form a large range; during the experiments a certain frequency produced also a 4th subharmonic (shown in vertical shading in Fig. 1) in addition to the 2nd subharmonic.

If the loudspeaker is subjected to an alternating voltage with a constant amplitude, but with a frequency continuously varying it will produce subharmonics within the ranges already mentioned. As the subharmonic, however, needs a certain time in which to rise, the change of frequency should not be too rapid. This is illus-

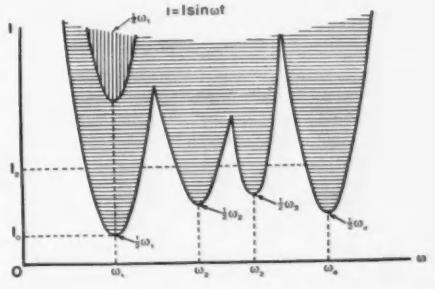


FIG. 1. Schematic representation of the occurrence of subharmonics. The system is being acted upon by an $AC: i = I \sin \omega t$.

* Ph.D., Professor Royal Technical College.

[1] P. O. Pedersen, *Ingeniøren*, p. 31, Copenhagen, January 28, 1933. This article gives a short summary of the experiments in question and the theoretical results attained. A very short note has appeared in Wireless Eng. and Exp. Wireless **10**, 313, June (1933). A full account of our work has been given in: P. O. Pedersen, *Subharmonics in forced oscillations in dissipative systems*, Ingeniørvidenskabelige Skrifter A No. 35 Copenhagen, 1933 (G. E. C. Gad). Referred to as "A 35." After the completion of our work a short report by F. R. W. Strafford has appeared in Wireless Eng. and Exp. Wireless **10**, 141, March (1933). It appears that Mr. Strafford has likewise observed second subharmonics by experiments with a moving-coil loudspeaker. No explanation of the occurrence of these subharmonics is, however, offered. N. W. McLachlan, l.c. p. 204, and H. F. O. Benecke, l.c. p. 257, seek the explanation of the subharmonics in "*mathieu* oscillations." This question is discussed further in Section 5.

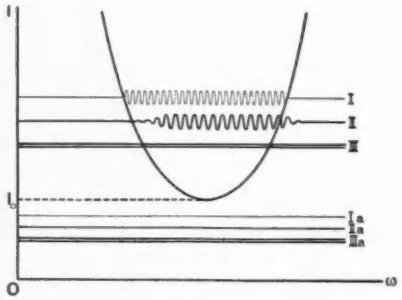


FIG. 2. Schematic representation of the occurrence of subharmonics. The lines I and Ia correspond to a very slowly varying frequency, the lines II, IIa to medium rapid, and the double lines III, IIIa to very rapid variations of frequency.

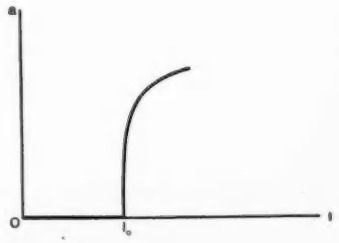


FIG. 3. The amplitude a of the subharmonic as a function of the amplitude I of the actuating force.

trated in Fig. 2 which shows how the growth of the subharmonic depends on the rapidity with which the frequency is changed. If the change of frequency is exceedingly slow and the corresponding amplitude exceeds I_0 the subharmonic will follow line I. If the change is exceedingly quick the subharmonic will not be left sufficient time to rise to a finite intensity (line III). Line II shows the result of moving through the particular range with a moderate rapidity. If $I<I_0$ no subharmonics are produced (lines Ia, IIa and IIIa).

Fig. 3 shows the relations between the amplitude a of the subharmonic and the amplitude I of the actuating force. When $I<I_0$, I_0 being the critical amplitude, then $a=0$; if $I>I_0$, a will increase from 0 to a finite and often comparatively high value, and the rate of this growth increases as I exceeds I_0. The steepness of the curve at the critical point is so considerable that it is often found impossible to adjust the intensity of the subharmonic at a comparatively low value. In a particular instance no subharmonics were produced when a current of 25.1 ma was used, while 25.2 ma had the effect of producing in a few seconds a subharmonic of a comparatively great strength, only slightly less than that produced by increasing further the current to 30 ma.

The subharmonic may be brought to disappear by placing a finger on the extreme edge of the loudspeaker diaphragm so as to give a slight damping effect, while this hardly influences the strength of the primary tone. If it is attempted to cut off the subharmonic through a similar damping of the movement of the moving coil, this will only be found possible through exercising a pressure so great that the strength of the primary tone is somewhat reduced. This may lead us to the deduction that the phenomenon of the subharmonic in this case largely depends upon a resonance of the diaphragm. The even distribution of the frequencies at which subharmonics are produced falls exactly in line with this assumption.

A direct verification of these resonances was attempted through the drawing-up of graphs for the impedance of the loudspeaker as a function of the frequency. These graphs also show with certainty irregularities for the frequencies corresponding to the note of certain of the subharmonics produced.

It appears from the following that the subharmonic really is a *harmonic* subnote with the frequency $\frac{1}{2}f$.

2. General Remarks

Harmonics in forced oscillations are, unfortunately, quite well-known phenomena which leave their undesired imprint upon practically all mechanical-electrical reproductions of speech, music, etc. The importance of these means of reproduction is gradually increasing very much and this has led to exhaustive enquiries into the whole question of the appearance of (higher) harmonics and their abolishment in recording devices, amplifiers, lines, loudspeakers, etc. The literature dealing with these subjects has greatly increased during the last few years.

As far as I know the question of subharmonics has, however, not been taken up previously, apart from certain cases where the system in itself contained sources of energy and was ar-

ranged so as to act as a generator of oscillations with the frequency of the subharmonic, thus reducing the impressed voltage to act as a kind of control of the low frequency generator oscillation (cf. The Multivibrator of Abraham and Bloch) a group which is referred to as relaxation oscillations by Balth. van der Pol.[2]

Most authors pass over this question in silence as they take it for granted that the occurrence of subharmonics is an impossibility. In attempting the approximate solution of the equation

$$d^2u/dt^2+n^2u+\alpha u^2=E\cos pt. \qquad \text{(a)}$$

Rayleigh[3] thus finds

$$u=e\cos pt-\alpha e^2/2n^2 - [\alpha e^2/2(n^2-4p^2)]\cos 2pt, \qquad \text{(b)}$$

where

$$e=E/(n^2-p^2). \qquad \text{(c)}$$

It will, however, be shown in the following that the solution of Eq. (a) may contain a second subharmonic.

H. v. Helmholtz,[4] and C. Schäfer[5] arrive at results in agreement with Rayleigh's.

G. Duffing[6] tacitly takes it for granted that subharmonics cannot appear. Several of his solutions for forced oscillations are thus only stable for small amplitudes and the solutions (reference 6, pp. 52–60) are unstable even at exceedingly small amplitudes. In the latter case subharmonics would always appear when the natural frequency of the oscillating circuit approaches the value $\frac{1}{2}f$; see Eq. (3.30), § 3.[7]

We have found only one author who openly denies the existence of subharmonics: R. Rüdenberg[8] expresses the following views upon the solution of the equation

$$d^2y/dt^2+x(y)=z(\omega t).$$

Wenn die Störungsfunktion z harmonischem Verlauf mit der Frequenz ω besitzt, so wird im stationären Zustand auch x und y periodisch mit der Frequenz ω verlaufen.

We are thus faced with the problem of how these subnotes are produced. In the following we shall endeavor to contribute towards the solution of this problem.

Mechanical and electrical systems will chiefly be dealt with as a whole, and we are chiefly devoting our attention to systems with one degree of freedom.[9] Furthermore, only the forced periodical oscillations are taken into account; while we are not considering the free oscillations of the system as these fade away in time by positive attenuation and rise infinitely in the case of negative attenuation.

It is well known that in systems whose movements are determined by linear differential equations with constant coefficients the forced oscillations will in all cases be periodic and with the same frequency as the moving force. And there is no other solution of the steady state problem. Neither higher harmonics nor subharmonics can possibly exist in this case.

The cases in which subharmonics may exist must therefore be looked for in systems whose movements are determined by differential equations which are either nonlinear or linear with coefficients which are functions of the independent variable, the time.

It is also well known, that nonlinear systems, e.g., systems in which the stiffness depends upon the displacement, when subjected to a periodic force $F_0\sin\omega t$, will perform a periodic displacement containing a fundamental oscillation with the circular frequency ω and a generally infinite series of higher harmonics.

The question is now: can the forced oscillations of such a system when acted upon by the pure

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[2] Balth. van der Pol, Phil. Mag. **2**, 978 (1926); Zeits. f. Hochfreq. Technik. **28**, 178 (1926); **29**, 114 (1927); 3rd International Congress of Technical Mechanics, Stockholm, 1931, **III**, p. 178.

Ph. le Corbeiller, *Les systèmes autoentretenus et les oscillations de relaxation*, Hermann et Cie., Paris, 1931.

[3] Rayleigh, *Theory of Sound* (2nd edition). Vol. I, §68, London, 1894.

[4] H. v. Helmholtz, *Die Lehre von den Tonempfindungen* (5th edition), pp. 650–652, Braunschweig, 1896.

[5] C. Schäfer, *Einführung in die theoretische Physik*, Vol. I, pp. 144–146, Leipzig, 1914.

[6] G. Duffing, *Erzwungene Schwingungen bei veränderlicher Eigenfrequenz und ihre technische Bedeutung*, Sammlung Vieweg, No. 41–42, Braunschweig, 1918.

[7] In Handb. d. Physik **8**, Akustik, p. 43 *et seq.* (Berlin, 1927) and in Handb. d. Exp. physik **17**, No. 2, Technische Akustik, No. 1, p. 10 *et seq.* (Leipzig, 1934) the problem is likewise passed in silence.

[8] R. Rüdenberg, *Einige unharmonische Schwingungsformen mit grosser Amplitude*, Zeits. f. ang. Math. u. Mech. **3**, particularly p. 463 (1923).

[9] For systems with more degrees of freedom see "A 35."

harmonic force $F_0 \sin \omega t$ also contain the second subharmonic with circular frequency $\frac{1}{2}\omega$ and possibly its higher odd harmonics besides the fundamental oscillations with the circular frequency ω and its higher harmonics.

It is this question which we shall investigate in the following for systems with one degree of freedom.

3. The Parameters Dependent upon the Displacement

Systems with one degree of freedom

The displacement x of a system with one degree of freedom is determined by the equation:

$$md^2x/dt^2+rdx/dt+sx=F \sin \omega t, \quad (3.01)$$

and if the coefficients m, r, s and F are constant the well-known solution of (3.01) is

$$x=A_1 \sin \omega t+B_1 \cos \omega t, \quad (3.02)$$

where
$$A_1=\frac{s-m\omega^2}{(s-m\omega^2)^2+r^2\omega^2}\cdot F$$

and
$$B_1=\frac{-r\omega}{(s-m\omega^2)^2+r^2\omega^2}\cdot F \quad (3.03)$$

(see Fig. 4).

The parameters of the systems will, however, in many practical cases be dependent upon the displacement, and this is always the case with the mechanical systems as Hooke's law only applies with approximation; the parameter s should really be replaced by $s_0(1+s(x))$ where $s(x)$ is a function of the displacement x and goes towards zero as x approaches this value. With regard to the resistance coefficient it is even frequently the case that only a very rough approximation is obtained where constant value is used. It is rather different with the electrical systems as in many cases, even with very considerable amplitudes of oscillation, the parameters (L, R, C) will retain their values practically unchanged. There are, however, electrical systems to which this does not apply, for instance in the case of conductors with a non-linear characteristic, i.e., conductors to which Ohm's law does not apply, e.g., rectifiers, coils with iron cores, etc.

Finally the factor F, which is found on the right-hand side of the Eq. (3.01) will in many cases be dependent on the displacement. This is, for instance, the case when an iron armature or a current carrying coil moves within an inhomogenous magnetic field. Although more or less pronounced this is the case with practically all acoustical reproducing devices. Also with regard to these, F could in many cases be replaced by $F_0(1+F(x))$ where $F(x)\to 0$ when $x\to 0$.

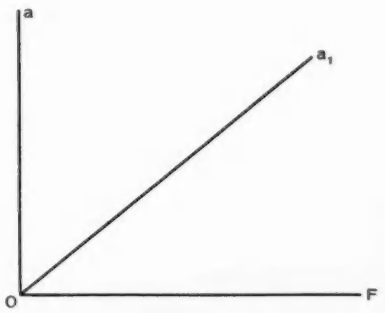

Fig. 4. Constant parameters. Force $F\cdot\sin \omega t$, displacement $x=A_1 \sin \omega t-B_1 \cos \omega t$; $a_1=(A_1^2+B_1^2)^{\frac{1}{2}}$.

If the constant parameters of the Eq. (3.01) were exchanged by

$$m=m_0(1+\mu_1 x+\mu_2 x^2+\cdots),$$
$$r=r_0(1+\beta_1 x+\beta_2 x^2+\cdots),$$
$$s=s_0(1+\alpha_1 x+\alpha_2 x^2+\cdots)$$

and
$$F=F_0(1+g_1 x+g_2 x^2+\cdots), \quad (3.04)$$

the Eq. (3.01) would represent with good approximation most of the mechanical and electrical systems used in practice. The equation is, however, no longer linear and in attempting to solve it we find great, if not unsurmountable, difficulties. It is, however, possible to gather some information about the forced oscillations of a system of this nature without too much difficulty. For purposes of simplification we will confine ourselves chiefly to the mechanical systems, but it should be noted that exactly the same conditions apply to the electrical systems.

It is easily ascertained that with very small forces and correspondingly small displacements all the parameters will very nearly assume their (constant) zero-values, i.e.,

$$m\to m_0, \quad r\to r_0, \quad s\to s_0 \quad \text{and} \quad F\to F_0$$

so that as long as the amplitudes are very small

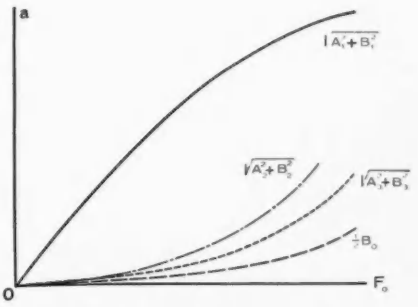

FIG. 5. Variable stiffness. No second subharmonic. Force $F_0 \sin \omega t$, displacement

$$x = A_1 \sin \omega t + A_2 \sin 2\omega t + A_3 \sin 3\omega t + A_4 \sin 4\omega t + \cdots + \tfrac{1}{2}B_0 + B_1 \cos \omega t + B_2 \cos 2\omega t + B_3 \cos \omega 3t + B_4 \cos 4\omega t + \cdots .$$

the forced oscillations will appear as if all the parameters were constant. In this case, the forced oscillations will thus all consist of purely sinusoidal oscillations with the frequency $f = \omega/2\pi$ of the fundamental oscillation and with amplitudes proportional to the amplitude of the motive force, F_0. We must therefore have

$$a_2/a_1 \to 0, \quad a_3/a_1 \to 0, \quad a_4/a_1 \to 0, \quad \cdots \quad \text{when} \quad a_1 \to 0 \qquad (3.05)$$

or the ratio between the amplitudes of the harmonics $a_2 = (A_2^2+B_2^2)^{\frac{1}{2}}$, $a_3 = (A_3^2+B_3^2)^{\frac{1}{2}}, \cdots$ and the amplitude $a_1 = (A_1^2+B_1^2)^{\frac{1}{2}}$ of the fundamental oscillation goes towards zero when a_1 goes towards zero, see Fig. 5, which corresponds to variable stiffness.

In order to simplify the following considerations we shall begin by considering the case of linearly variable force. In this case we have

$$m_0 d^2x/dt^2 + r_0 dx/dt + s_0 x = F_0(1+g_1 x) \sin 2\omega t, \qquad (3.06)$$

the circular frequency of the vibromotoric force we assume to be 2ω.

We assume, that there exists a periodic solution of the Eq. (3.06) with the period $\tau = 2 \cdot 2\pi/2\omega$. The displacement may then be written:

$$x = A_1 \sin \omega t + A_2 \sin 2\omega t + A_3 \sin 3\omega t + A_4 \sin 4\omega t + \cdots + \tfrac{1}{2}B_0 + B_1 \cos \omega t + B_2 \cos 2\omega t + B_3 \cos 3\omega t + B_4 \cos 4\omega t + \cdots . \qquad (3.07)$$

If we use the following abbreviations

$$2\gamma = \frac{r_0}{m_0}, \quad \omega_0^2 = \frac{s_0}{m_0}, \quad G_0 = \frac{F_0}{m_0}, \quad p_0 = \frac{\gamma}{\omega_0} = \frac{r_0}{2(m_0 s_0)^{\frac{1}{2}}}, \quad h_0 = \frac{g_1 G_0}{2\omega_0^2} \text{ and } \xi = \frac{\omega}{\omega_0}, \qquad (3.08)$$

the Eq. (3.06) reduces to

$$d^2x/dt^2 + 2\gamma dx/dt + \omega_0^2 x = G_0(1+g_1 x) \sin 2\omega t. \qquad (3.09)$$

It is easily found that in order to get for (3.09) the solution (3.07) the coefficients (A, B) must satisfy the following system of equations:

$$\left.\begin{aligned} \tfrac{1}{2}B_0 - h_0 A_2 &= 0, \\ (1-\xi^2)A_1 - 2p_0\xi B_1 - h_0(B_1 - B_3) &= 0, \\ (1-\xi^2)B_1 + 2p_0\xi A_1 - h_0(A_1 + A_3) &= 0, \\ (1-4\xi^2)A_2 - 4p_0\xi B_2 - h_0(B_0 - B_4) &= G_0/\omega_0^2, \\ (1-4\xi^2)B_2 + 4p_0\xi A_2 - h_0 A_4 &= 0, \\ (1-9\xi^2)A_3 - 6p_0\xi B_3 - h_0(B_1 - B_5) &= 0, \\ (1-9\xi^2)B_3 + 6p_0\xi A_3 + h_0(A_1 - A_5) &= 0, \\ (1-16\xi^2)A_4 - 8p_0\xi B_4 - h_0(B_2 - B_6) &= 0, \\ (1-16\xi^2)B_4 + 8p_0\xi A_4 + h_0(A_2 - A_6) &= 0, \\ \cdots\cdots\cdots\cdots\cdots\cdots & \end{aligned}\right\} \begin{matrix} 0 \\ 1 \\ 1 \\ 2 \\ 2 \\ 3 \\ 3 \\ 4 \\ 4 \\ \cdots \end{matrix} \qquad (3.10)$$

Closer examination will make it clear that the above system of linear equations divides itself into the two following linear and independent groups

$$\left.\begin{aligned} \tfrac{1}{2}B_0 - h_0 A_2 &= 0, \\ (1-4\xi^2)A_2 - 4p_0\xi B_2 - h_0(B_0 - B_4) &= G_0/\omega_0^2, \\ (1-4\xi^2)B_2 + 4p_0\xi A_2 - h_0 A_4 &= 0, \\ (1-16\xi^2)A_4 - 8p_0\xi B_4 - h_0(B_2 - B_6) &= 0, \\ (1-16\xi^2)B_4 + 8p_0\xi A_4 + h_0(A_2 - A_6) &= 0, \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \end{aligned}\right\} \quad (3.11)$$

$$\left.\begin{aligned} (1-\xi^2)A_1 - 2p_0\xi B_1 - h_0(B_1 - B_3) &= 0, \\ (1-\xi^2)B_1 + 2p_0\xi A_1 - h_0(A_1 + A_3) &= 0, \\ (1-9\xi^2)A_3 - 6p_0\xi B_3 - h_0(B_1 - B_5) &= 0, \\ (1-9\xi^2)B_3 + 6p_0\xi A_3 + h_0(A_1 - A_5) &= 0, \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \end{aligned}\right\} \quad (3.12)$$

It will be noticed that the group (3.11) consists of the Eqs. (0), (2), (4), (6), etc., and that all the coefficients have even indices. In this particular case with a linear dependency between the amplitude of displacement and the amplitude of the motive force the Eqs. (3.11) are actually identical with the equation which would result from putting the displacement equal to

$$\begin{aligned} x = {} & A_2 \sin 2\omega t + A_4 \sin 4\omega t \\ & + A_6 \sin 6\omega t + \cdots \\ & + \tfrac{1}{2}B_0 + B_2 \cos 2\omega t + B_4 \cos 4\omega t \\ & + B_6 \cos 6\omega t + \cdots \end{aligned} \quad (3.13)$$

leaving out the possibility of subharmonics. The other group of Eqs. (3.12) consists of the Eqs. (1), (3), (5), etc., and in the present case all those equations only contain coefficients (A, B) with odd indices. It will further be seen that the Eqs. (3.12) are homogeneous, consequently they are satisfied by putting all coefficients with an odd index equal to zero, in other words we revert to the solution for x given in (3.13).

The linearity and the independency of the Eqs. (3.11) and (3.12) is due to the fact that all the parameters of the system are constant except the motive force which simply depends *linearly* upon the displacement. If a linear dependency is introduced in the other parameters, or if it is assumed that one or several of the parameters depend upon the second or a higher power of the displacement, the coefficient equations become of a higher degree and both types of coefficients are then generally to be found in all the equations. However, in the particulars decisive for the treatment of our problem the equations may be arranged in two such groups of which one is analogous with (3.11) and the other with (3.12).

All equations in the group analogous with (3.11) will contain on the left side at least one term having a coefficient with an even index as a factor, and also at least one term which does not contain a coefficient with an odd index as a factor. All the terms on the right-hand side will be zero with the exception of one equation which on its right-hand side has the constant term.

Similarly *all* equations analogous with the group (3.12) will have zero on the right-hand side and all terms on the left-hand side will contain at least one of the coefficients with odd indices as a factor.

The correctness of these postulates is a simple consequence of the elementary trigonometrical rules of calculation ($\sin a \cdot \sin b = \frac{1}{2}\cos(a-b) - \frac{1}{2}\cos(a+b)$, etc.) and is easily proved. A similar division into two groups of equations

may be made when trying to ascertain the conditions of the occurrence of subharmonics with the frequency $\frac{1}{2}f$, $\frac{1}{4}f$, etc.

Even a more complicated structure of the parameters will thus not eliminate the difficulty that the set of equations of the coefficients with odd indices is satisfied by putting all these coefficients equal to zero, whereby the looked-for subharmonic and its higher odd harmonics disappear.

The series (3.13) for the displacement x will thus always be a solution of (3.09) provided the coefficients A_2, A_4, $\cdots$ B_0, B_2, B_4, satisfy the Eqs. (3.11).

The question is now: has the Eq. (3.09) also a solution of the form (3.07) with finite values of the coefficients A_1, B_1, A_3, B_3, $\cdots$?

In order to make this investigation as simple as possible we will commence by assuming the system to be tuned to the frequency $\frac{1}{2}f=\frac{1}{2}\cdot 2\omega/2\pi$, i.e., $\omega=\omega_0$ or $\xi=\xi_0=1$. It is then to be expected that the conditions for the occurrence of a subharmonic with the frequency $\frac{1}{2}f$ will be as favorable as possible. The Eqs. (3.11) and (3.12) may then be written as follows:

$$\begin{aligned}
\tfrac{1}{2}B_0-h_0A_2 &= 0,\\
3A_2+4p_0B_2+h_0(B_0-B_4) &= G_0/\omega_0^2,\\
-3B_2+4p_0A_2-h_0A_4 &= 0,\\
15A_4+8p_0B_4+h_0(B_2-B_6) &= 0,\\
-15B_4+8p_0A_4+h_0(A_2-A_6) &= 0,\\
&\cdots\cdots\cdots\cdots
\end{aligned} \tag{3.14}$$

$$\begin{aligned}
2p_0B_1+h_0(B_1-B_3) &= 0,\\
2p_0A_1-h_0(A_1+A_3) &= 0,\\
8A_3+6p_0B_3+h_0(B_1-B_5) &= 0,\\
-8B_3+6p_0A_3+h_0(A_1-A_5) &= 0,\\
&\cdots\cdots\cdots\cdots
\end{aligned} \tag{3.15}$$

We will further assume the motive force to be so small and the damping so slight that both $|h_0|\ll 1$ and $p_0\ll 1$. We may then expect a reasonable approximation even if we include only a few terms of the series (3.07).

Under these conditions the Eqs. (3.14) will give us the following first approximation of the coefficients with even indices:

$$A_2=-\frac{G_0}{3\omega_0^2}=-\frac{F_0}{3s_0},$$

$$B_2=-\frac{4}{9}\frac{p_0}{\omega_0^2}G_0=-\frac{2}{9}\frac{r_0}{s_0(m_0s_0)^{\frac{1}{2}}}F_0$$

and

$$B_0=-\frac{2h_0}{3\omega_0^2}G_0=-\frac{1}{3}\frac{g_1}{12}\frac{F_0^2}{s_0^2}, \tag{3.16}$$

while all the other coefficients with even indices are put equal to zero.

We will then proceed to the Eqs. (3.15). Assuming that with sufficient approximation all the coefficients with an index exceeding 3 may be put equal to zero, then in order to make possible other solutions of the four top equations than the trivial one where all four coefficients A_1, B_1, A_3 and B_3 are zero, it is necessary that their determinant should be equal to zero, or

$$\Delta=\begin{vmatrix} 0 & 2p_0+h_0 & 0 & -h_0\\ 2p_0-h_0 & 0 & -h_0 & 0\\ 0 & h_0 & 8 & 6p_0\\ h_0 & 0 & 6p_0 & -8 \end{vmatrix}=0, \tag{3.17}$$

or

$$\Delta=h_0^4-(64+12p_0^2)h_0^2+256p_0^2+144p_0^4=0, \tag{3.18}$$

which gives

$$h_0^2=32+6p_0^2\overset{(+)}{-}(1024+128p_0^2-108p_0^4)^{\frac{1}{2}}\cong 4p_0^2+(27/16)p_0^4, \tag{3.19}$$

or

$$h_0\cong\pm(2p_0+(27/64)p_0^3),$$

which may also be written:

$$h_0/2p_0=\frac{g_1F_0}{\omega_0}/2r_0\cong\pm(1+(27/128)p_0^2). \tag{3.20}$$

Under the assumption $p_0\ll 1$, the condition (3.20) may be reduced to

$$\pm h_0=2p_0=(3/2)g_1|A_1| \quad \text{or}$$

$$\pm g_1F_0=4\gamma\omega_0m_0=2r_0\omega_0. \tag{3.21}$$

The corresponding values of the four coefficients will for $h_0=+2p_0$, be:

A_1 arbitrary, $B_1\cong 0$, $A_3\cong 0$ and $B_3\cong 0$.

Assuming that the relation (3.21) is fulfilled, we have consequently a possible solution of the form:

$$x=-\frac{g_1G_0^2}{6\omega_0^4}+A_1\sin\omega t-\frac{G_0}{3\omega_0^2}\sin 2\omega t$$
$$-\frac{4}{9}\frac{\gamma}{\omega_0^3}G_0\cos 2\omega t. \quad (3.22)$$

The average power, P_m supplied to the system is defined by

$$P_m=\frac{1}{2}\frac{2\omega}{2\pi}\int_0^{2\cdot 2\pi/2\omega}\frac{dx}{dt}m_0G_0(1+g_1x)\sin 2\omega t\,dt$$
$$=\frac{1}{4}\omega m_0g_1G_0A_1^2+\frac{4}{9}\frac{\gamma\omega m_0}{\omega_0^3}G_0\left(G_0-\frac{g_1^2G_0^3}{6\omega_0^4}\right)$$
$$\leqq\frac{\omega_0}{4}m_0g_1G_0A_1^2+\frac{4}{9}\frac{\gamma m_0}{\omega_0^2}G_0^2=P_i+P_1, \quad (3.23)$$

as it was assumed that $g_1^2G_0^2/6\omega_0^4\ll 1$, while

$$P_i=\tfrac{1}{4}\omega_0m_0g_1G_0A_1^2$$
and $$P_1=(4/9)\gamma m_0/\omega_0^2\cdot G_0^2. \quad (3.24)$$

P_1 exactly covers the loss of energy necessary to maintain the oscillation $-G_0/3\omega_0^2\cdot\sin 2\omega t$, as we have

$$\tfrac{1}{2}(2\omega/3\omega_0^2\cdot G_0)^2r_0=(4/9)\gamma m_0/\omega_0^2\cdot G_0^2=P_1.$$

According to our assumptions the power necessary for maintaining the oscillation

$$-(4/9)\gamma/\omega_0^3\cdot G_0\cos 2\omega t$$

is negligible compared with P_1.

It now only remains to examine the power W_i necessary for maintaining the oscillation $A_1\sin\omega t$. We have

$$W_i=\tfrac{1}{2}r_0(\omega A_1)^2=\tfrac{1}{2}r_0\omega_0^2A_1^2=\gamma m_0\omega_0^2A_1^2. \quad (3.25)$$

It is easily seen that

$$P_i\gtreqless W_i \text{ for } |g_1G_0|\gtreqless 4\gamma\omega_0 \text{ or } |h_0|\gtreqless 2p_0.^{10} \quad (3.26)$$

For $|h_0|=2p_0$ the power P_i just suffices to cover the loss of energy sustained by maintaining the subharmonic. $|h_0|=2p_0$ was, however, the mathematical condition for maintaining the subharmonic in the particular case where all the amplitudes of the oscillations are small. In this case, as it has already been shown, the amplitude of the subharmonic is quite arbitrary.

If $|h_0|<2p_0$ then a subharmonic started one way or another will fade away because the supply of energy will not be sufficient to make up for the loss of energy sustained through its maintenance. If, however, $|h_0|>2p_0$ the amplitude of a subharmonic started will increase, as the energy supplied more than covers the loss sustained through its maintenance. When $|h_0|>2p_0$ and when as a consequence hereof the amplitude of the subharmonic is comparatively large, it is no longer possible to put all the other coefficients with odd indices equal to zero and only retain A_1 and B_1. The more $|h_0|$ exceeds $2p_0$, the more terms it will be necessary to include. For instance if we take the slightly greater value of $|h_0|$ from (3.20):

$$h_0\leqq 2p_0(1+(27/128)p_0^2), \quad (3.27)$$

we find correspondingly

$$B_1=(27/224)p_0A_1,\quad A_2=-(27/128)p_0^2A_1,$$
and $$B_3=(27/112)p_0A_1, \quad (3.28)$$

while all other coefficients with odd indices are put equal to zero.

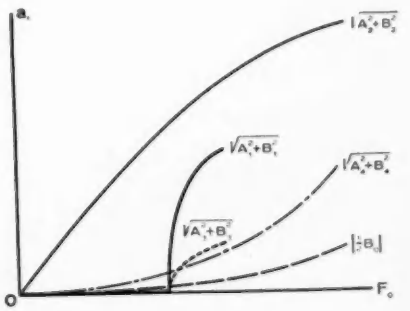

FIG. 6. Schematical representation of the dependency of the amplitudes $a_r=(A_r^2+B_r^2)^{\frac{1}{2}}$ upon the amplitude of the motive force in case of a system with either variable stiffness or variable motive force. The second subharmonic present. The motive force: $F_0\sin 2\omega t$.

[10] For $h_0=-2p_0$ we find $A_1\leqq 0$ and B_1 arbitrary, and in this case we have the following expression for $P_i=-\frac{1}{4}\omega_0m_0g_1G_0B_1^2$; as, however, g_1G_0 in this case is negative, P_i will be positive, and the following considerations as to the energy will likewise apply.

For $|h_0|>2p_0$ it will thus be possible to obtain the subharmonic $A_1 \sin \omega t + B_1 \cos \omega t$ and a series of higher odd harmonics hereof since with an increasing force more and more significant terms are added to the series, compare Fig. 6.

The Eqs. (3.15) are satisfied in all cases when all coefficients with odd indices are zero. And this solution is stable for $|h_0|<2p_0$ but unstable for $|h_0|>2p_0$.

All the coefficients with odd indices put equal to zero represent an equilibrium, but in case A_1 or B_1 for one reason or another should have assumed even a very small but finite value, this oscillation and a part of its harmonics will for $|h_0|>2p_0$ rise to finite values which will depend upon the strength of the motive force and the parameters of the system.

The condition $|h_0|=2p_0$ only applies when $\xi=\xi_0=1$ but as shown by Fig. 1 the occurrence of subharmonics is not dependent hereon as it is also possible to produce subharmonics in a certain neighborhood of $\xi=1$. The correctness hereof soon becomes evident if we study more closely the Eqs. (3.12); we will find as a first approximation that the condition for the occurrence of subharmonics is that the determinant of the equations

$$\begin{aligned}(1-\xi^2)A_1-2p_0\xi B_1-h_0' B_1=0,\\ (1-\xi^2)B_1+2p_0\xi A_1-h_0' A_1=0,\end{aligned} \tag{3.29}$$

is equal to zero, i.e.,

$$\Delta=(1-\xi^2)^2+4p_0^2\xi^2-h_0'^2=0,$$

or

$$h_0'^2=(1-\xi^2)^2+4p_0^2\xi^2. \tag{3.30}$$

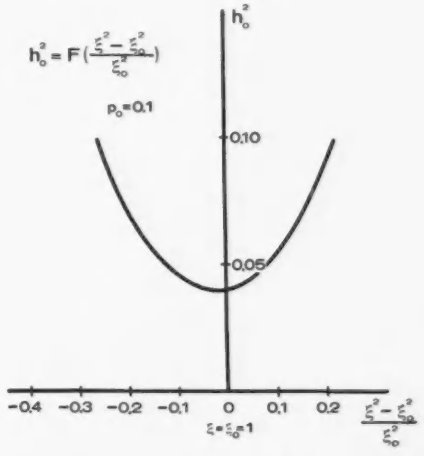

FIG. 7. h_0^2 as a function of $(\xi^2-\xi_0^2)/\xi_0^2=\xi^2-1$ for $p_0=0.1$, and in the case where the amplitude of the motive force is linearly dependent upon the displacement.

h_0^2 has its minimum value $4p_0^2(1-p_0^2)$ for $\xi^2=1-2p_0^2$. The point $(1, 4p_0^2)$ (which corresponds to $\xi=1$) on the curve $h_0^2=F(\xi^2)$ thus approaches closer to the minimum point the smaller p_0 is. Fig. 7 shows h_0^2 as a function of the "detuning" $(\xi^2-\xi_0^2)/\xi_0^2=\xi^2-1$.

We will now treat a case where the stiffness is linearly dependent upon the displacement:

$$d^2x/dt^2+2\gamma dx/dt+\omega_0^2(1+\alpha_1 x)x = G_0 \sin 2\omega t. \tag{3.31}$$

Inserting the expression for x given by the Eq. (3.07) in the Eq. (3.31) we arrive at the following two systems of equations for determining the coefficients

$$\begin{aligned}
&\tfrac{1}{2}B_0+\alpha_1(\tfrac{1}{4}B_0^2+\tfrac{1}{2}(A_1^2+B_1^2+A_2^2+B_2^2+A_3^2+B_3^2\cdots))=0,\\
&(1-4\xi^2)A_2-4p_0\xi B_2+\alpha_1(B_0A_2+A_1B_1-A_1B_3+B_1A_3\\
&\qquad -A_2B_4+B_2A_4-A_3B_5+B_3A_5\cdots)=G_0/\omega_0^2,\\
&(1-4\xi^2)B_2+4p_0\xi A_2+\alpha_1(B_0B_2-\tfrac{1}{2}A_1^2+A_1A_3+\tfrac{1}{2}B_1^2\\
&\qquad +B_1B_3+A_2A_4+B_2B_4+A_3A_5+B_3B_5\cdots)=0,\\
&(1-16\xi^2)A_4-8p_0\xi B_4+\alpha_1(B_0A_4+A_1B_3-A_1A_5+B_1A_3\\
&\qquad +B_1A_5+A_2B_2-A_2B_6+B_2A_6-A_3B_7+B_3A_7\cdots)=0,\\
&(1-16\xi^2)B_4+8p_0\xi A_4+\alpha_1(B_0B_4-A_1A_3+A_1A_5+B_1B_3+B_1B_5\\
&\qquad -\tfrac{1}{2}A_2^2+A_2A_6+\tfrac{1}{2}B_2^2+B_2B_6+A_3A_7+B_3B_7\cdots)=0,\\
&\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots
\end{aligned} \tag{3.32}$$

$$
\begin{aligned}
&(1-\xi^2)A_1-2p_0\xi B_1+\alpha_1(B_0A_1-A_1B_2+B_1A_2-A_2B_3\\
&\qquad +B_2A_3-A_3B_4+B_3A_4-A_4B_5+B_4A_5\cdots)=0,\\
&(1-\xi^2)B_1+2p_0\xi A_1+\alpha_1(B_0B_1+A_1A_2+B_1B_2+A_2A_3\\
&\qquad +B_2B_3+A_3A_4+B_3B_4+A_4A_5+B_4B_5\cdots)=0,\\
&(1-9\xi^2)A_3-6p_0\xi B_3+\alpha_1(B_0A_3+A_1B_2-A_1B_4+B_1A_2\\
&\qquad +B_1A_4-A_2B_5+B_2A_5-A_3B_6+B_3A_6\cdots)=0,\\
&(1-9\xi^2)B_3+6p_0\xi A_3+\alpha_1(B_0B_3-A_1A_2+A_1A_4+B_1B_2\\
&\qquad +B_1B_4+A_2A_5+B_2B_5+A_3A_6+B_3B_6\cdots)=0,\\
&\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots
\end{aligned}
\tag{3.33}
$$

If the coefficients (A, B) satisfy the Eqs. (3.32) and (3.33) then the series (3.07) for the displacement x will be a solution of the Eq. (3.31).

Supposing

$$|\alpha_1 x|\ll 1 \quad \text{and} \quad p_0=\gamma/\omega_0=r_0/2m_0\omega_0\ll 1,$$

the solution of the equations

$$
\begin{aligned}
&(1-4\xi^2)A_2-4p_0\xi B_2=G_0/\omega_0^2,\\
&4p_0\xi A_2+(1-4\xi^2)B_2=0,
\end{aligned}
\tag{3.34}
$$

will give a first approximation of the values of the amplitudes A_2 and B_2. We find

$$A_2=\frac{1-4\xi^2}{(1-4\xi^2)^2+16p_0^2\xi^2}\cdot\frac{G_0}{\omega_0^2}$$

and

$$B_2=\frac{-4p_0\xi}{(1-4\xi^2)^2+16p_0^2\xi^2}\cdot\frac{G_0}{\omega_0^2}, \tag{3.35}$$

while

$$A_2^2+B_2^2=\frac{1}{(1-4\xi^2)^2+16p_0^2\xi^2}\cdot\frac{G_0^2}{\omega_0^4}. \tag{3.36}$$

The top Eq. (3.32) gives the following approximation

$$\alpha_1 B_0\leqq-\alpha_1^2(A_2^2+B_2^2). \tag{3.37}$$

Under the same assumptions the two first equations in (3.33) may be reduced to

$$
\begin{aligned}
&[(1-\xi^2)-\alpha_1^2(A_2^2+B_2^2)-\alpha_1B_2]A_1\\
&\qquad +(\alpha_1A_2-2p_0\xi)B_1=0,\\
&(\alpha_1A_2+2p_0\xi)A_1+[(1-\xi^2)\\
&\qquad -\alpha_1^2(A_2^2+B_2^2)+\alpha_1B_2]B_1=0.
\end{aligned}
\tag{3.38}
$$

To solve these equations with regard to (A_1, B_1) the determinant of the Eqs. (3.38) must be equal to zero.

$$\Delta=[(1-\xi^2)-\alpha_1^2(A_2^2+B_2^2)]^2-\alpha_1^2B_2^2-\alpha_1^2A_2^2+4p_0^2\xi^2=0,$$

or

$$\alpha_1^4(A_2^2+B_2^2)^2-(3-2\xi^2)\alpha_1^2(A_2^2+B_2^2)+(1-\xi^2)^2+4p_0^2\xi^2=0. \tag{3.39}$$

According to the assumptions made we have $\alpha_1^4(A_2^2+B_2^2)^2\ll\alpha_1^2(A_2^2+B_2^2)$ and provided $3-2\xi^2>0$ we have

$$\alpha_1^2(A_2^2+B_2^2)\leqq\frac{(1-\xi^2)^2+4p_0^2\xi^2}{3-2\xi^2}.$$

Closer examination shows that $\alpha_1^2(A_2^2+B_2^2)$ becomes a minimum for $\xi^2\leqq 1-6p_0^2$ and the minimum value will be

$$[\alpha_1^2(A_2^2+B_2^2)]_{\min.}\leqq 4p_0^2(1-9p_0^2). \tag{3.40}$$

As long as $p_0\ll 1$ the minimum point of the curve $\alpha_1^2(A_2^2+B_2^2)=F(\xi^2)$ will thus fall near the point $(1,\ 4p_0^2)$. Fig. 8 shows the shape of $\alpha_1^2(A_2^2+B_2^2)$ as a function of the "detuning"

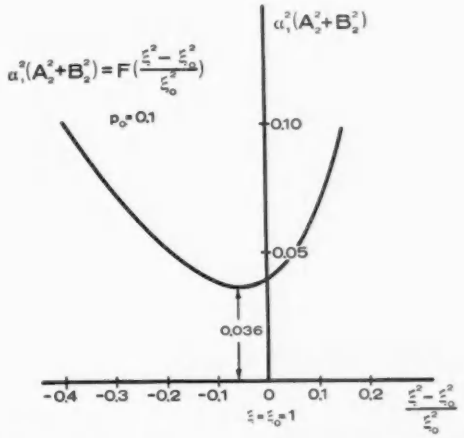

FIG. 8. $\alpha_1^2(A_2^2+B_2^2)$ as a function of (ξ^2-1) for $p_0=0.1$ and for linear dependence of the stiffness upon the displacement.

$(\xi^2-\xi_0^2)/\xi_0^2=\xi^2-1$ for a system with a damping corresponding to $p_0=0.1$.

It will be seen that the curve is only of interest within such ranges where the inequality

$$\alpha_1^2(A_2^2+B_2^2)\ll 1$$

is complied with as otherwise our calculations will be invalid. For $\xi=1$, i.e., for $\omega=\omega_0$ we have

$$\alpha_1^2(A_2^2+B_2^2)=4p_0^2, \tag{3.41}$$

as a condition for the occurrence of subharmonics. As our assumptions involve that $B_2/A_2 \ll 1$ we find the following approximate conditions:

$$\alpha_1^2 A_2^2 \leqq 4p_0^2 = r_0^2/m_0 s_0$$

or

$$\alpha_1 A_2 \leqq \pm 2p_0 = \pm r_0/(m_0 s_0)^{\frac{1}{2}}. \tag{3.42}$$

From the top equation (3.38) we get for $\xi=1$

$$\frac{A_1}{B_1}=\frac{\alpha_1^2(A_2^2+B_2^2)+\alpha_1 B_2}{\alpha_1 A_2-2p_0}, \tag{3.43}$$

while the Eq. (3.35) gives $|A_2/B_2|=3/4p_0$.

For $\alpha_1 A_2=-2p_0$ (α_1 positive) and as a consequence of (3.43) we have $|B_1/A_1|\ll 1$ while for $\alpha_1 A_2=+2p_0$ (α_1 negative) we have the ratio $|B_1/A_1|\gg 1$.[11]

[11] To $|\alpha_1 A_2|=|2p_0|$ corresponds $|\alpha_1 G_0|=3\omega_0 x_0/m_0$.

In order to investigate the power relations it is necessary to include one more term of the Eqs. (3.32); for the determination of A_2 and B_2 we then get instead of (3.34) the following Eqs. ($\xi=1$)

$$\begin{aligned} -3A_2-4p_0B_2&=(1/\omega_0^2)G_0,\\ 4p_0A_2-\ 3B_2\ &=\alpha_1 A_1^2, \end{aligned} \tag{3.44}$$

where we have assumed α_1 to be positive, hence $\alpha_1 A_2=-2p_0$, and $|B_1/A_1|\ll 1$. Consequently

$$A_2 \leqq -\frac{1}{9}\left(\frac{3}{\omega_0^2}G_0-2p_0\alpha_1 A_1^2\right),$$

$$B_2 \leqq -\frac{1}{9}\left(\frac{4p_0}{\omega_0^2}G_0+\frac{3}{2}\alpha_1 A_1^2\right),$$

and

$$A_2^2+B_2^2 \leqq \frac{1}{9}\left(\frac{1}{\omega_0^4}G_0^2+\frac{1}{4}\alpha_1^2 A_1^4\right) \leqq \frac{1}{9\omega_0^4}G_0^2. \tag{3.45}$$

Thus the presence of the oscillation $A_1 \sin \omega_0 t$ has but little influence upon the resultant amplitude $(A_2^2+B_2^2)^{\frac{1}{2}}$ of the fundamental oscillation (of angular frequency $2\omega_0$), but in accordance with (3.41) it has the effect of reducing $|A_2|$ and increasing $|B_2|$.

We therefore have an increase of the supplied power

$$P_m=-\tfrac{1}{2}\cdot 2\omega_0 F_0 B_2$$

$$=\frac{1}{9}\omega_0 F_0\left(\frac{4p_0}{\omega_0^2}G_0+\frac{3}{2}\alpha_1 A_1^2\right), \tag{3.46}$$

as we have assumed α_1 to be positive.

The maintenance of the fundamental oscillation requires the power

$$W_1=\tfrac{1}{2}(2\omega_0)^2(A_2^2+B_2^2)r_0$$

$$\leqq \frac{4p_0F_0^2}{9\omega_0 m_0}=\omega_0 F_0\frac{4}{9}\frac{p_0}{\omega_0^2}G_0. \tag{3.47}$$

The subharmonic $A_1 \sin \omega_0 t$ requires the power

$$W_1=\tfrac{1}{2}\omega_0^2 A_1^2 r_0=m_0 p_0 \omega_0^3 A_1^2$$

$$=-m_0\omega_0^3\frac{\alpha_1}{2}A_2 A_1^2=\frac{1}{6}\omega_0 F_0 \alpha_1 A_1^2, \tag{3.48}$$

as $\frac{1}{2}\alpha_1 A_2=-p_0$.

From the Eqs. (3.46–3.48) follows $W_1+W_i=P_m$, a relation which is, and of necessity must be, fulfilled in the present case where the energy is supplied *exclusively* by means of the motive force $F_0 \sin 2\omega_0 t$ and the entire loss of energy is bue to the constant resistance r_0, in that the variable stiffness involves no exchange of energy detween the oscillating system and its surroundings.

Similar results are arrived at when α_1 is negative, i.e., for $\alpha_1 A_2 = 2p_0$.

The dependency of the fundamental oscillation and of the different harmonics upon the amplitude of the motive force in the case of variable stiffness is similar to that in the case of variable force shown in Fig. 6, and the general features of this figure are also in complete agreement with our experimental results. The observed subharmonics may therefore be explained equally well either as depending upon "variable force," Eq. (3.06), or upon "variable stiffness," Eq. (3.31).

Subharmonics may also occur if both the motive force and the stiffness depend upon the displacement. The dependency of the motive force or the stiffness upon the displacement need not be linear, but may, for instance, be given by

$$x = x_0(1+\alpha_1 x+\alpha_2 x^2+\cdots) = x_0(1+\alpha(x))$$
and (or) (3.49)
$$F = F_0(1+g_1 x+g_2 x^2+\cdots) = F_0(1+g(x)).$$

It may be proved, however, that subharmonics cannot appear when the parameters m, r, s and F all are even functions of the displacement.[12]

It may also be proved that only second subharmonics can exist in systems with only one degree of freedom.[13] In two coupled systems there is, under very favorable conditions, a possibility for the appearance of a fourth subharmonic.[14]

With regard to the influence of "variable mass" or "variable resistance" on the appearance of subharmonics the reader is referred to "A 35," pp. 32–35.

[12] "A 35," p. 61.
[13] "A 35," Sect. 10, p. 50.
[14] "A 35," Sect. 11, p. 62.

[To be concluded in the July issue]

What is Measured in Sound Absorption Measurements

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THE committee on nomenclature of the American Standards Association has defined the acoustic absorptivity of a surface as one minus the reflectivity of that surface, and reflectivity is defined as the ratio of the rate of flow of sound energy reflected from the surface to the incident rate of flow, all possible directions of incident flow being assumed equally probable and the value obtained "applies to a portion of an infinite surface, thus eliminating edge effects."

One recognizes that back of this definition of acoustical reflectivity is the presumably analogous case of optical reflectivity in which an approximation to an infinite surface measured in wavelengths of light is readily obtained and the laws of geometrical optics, uncomplicated by wave phenomena, i.e., diffraction and interference, are realized. One recognizes that also involved in this definition are the fundamental assumptions of the reverberation theory; namely, that of a diffuse distribution in a room of the sound energy in all stages of the decay, and that of the strict proportionality between the equivalent absorption of a surface and its area.

Now both of these assumptions go back to the fact that in deriving any of the now numerous reverberation formulas we ignore the wave nature of sound and treat small elements of the sound energy as though they were particles of a gas, moving independently of each other, and in so doing we employ both the concepts and terminology of the kinetic theory of gases to account for the approximately logarithmic decay of reverberant sound. Following Wallace Sabine's experimental deduction of the simple relation, $T_0=KV/\alpha S$, in which V is the total volume, S the total surface of a room and α is the mean value of the absorptivity, Franklin first, and subsequently Jaeger and others, deduced Sabine's laws by treating the problem of the behavior of sound in a room as analogous to that of a swarm of particles moving independently of each other and losing energy only upon impact and reflection at the bounding surfaces. With such a picture, the decay process is strictly logarithmic and the equivalent absorption of a surface directly proportional to its area. If we are dealing with rooms whose linear dimensions are large in comparison with the wavelength of sound and in which the absorption is fairly uniformly distributed over the bounding surfaces this point of view is legitimate enough. The actual conditions depart more and more widely from our theoretical picture as the dimensions of the room in wavelengths decrease and the absorption is localized in small areas.

As a practical matter, what we actually want in the absorption coefficient of a material is a numerical value such that when multiplied by the surface area of the material that is present in a room of any size, the product will give us the contribution of that surface to the total equivalent absorption of the room as computed by reverberation theory from the measured rate of decay of reverberant sound in that particular room. There are two questions that immediately arise: First, "Is there any such single value?" and second, "How are we going to find it?"

Up to the present, we have proceeded on the assumption that such a coefficient exists.

Our procedure in finding it by reverberation chamber measurements has been to measure by one of a number of different methods the rate of decay without and with a known area of the test sample present. The total equivalent absorptions under the two conditions are obtained from the relation,

$$a=(0.92V/c)m,$$

where a is the total equivalent absorption, V the volume of the chamber, c the velocity of sound, and m is the decibel drop per second. The experimentally determined quantity m is the decibel rate of decrease of the intensity averaged throughout the entire volume of the room, and assumed to be constant during the decay period.

Dividing the difference in total equivalent absorption by the area of the test sample that produces the difference we arrive at what we call the absorption coefficient of the material. Note that in this last step we assume strict

proportionality between the equivalent absorption and the area of the material.

Now while it is questionable whether the proportionality factor between area and total equivalent absorption is the absorptivity as defined by the American Standards Association, yet this factor, if constant for all areas in all rooms, would give us a figure of merit of the material which could be used in computing the effect on the rate of decay produced by any area of that material in any room, which, from a practical standpoint, is what we want.

Unfortunately the dimensions of our reverberation chamber and of our test samples are not large in comparison with the wavelength for low pitched sounds. The largest reverberation chamber, that at the Bureau of Standards, is $25\times30\times20$ ft., or approximately $12\times15\times10$ wavelengths of a 500 cycle tone. Ordinarily, test samples are of the order of 8×9 feet, or $4\times4\frac{1}{2}$ wavelengths. As a result the statistical conception of freely moving waves becomes an inadequate picture of what really takes place and the phenomenon is complicated by interference and diffraction effects.

Taking these into account, we are led to the theoretical treatment of the modes of free vibration of a three-dimensional continuum bounded by reflecting walls given by Lord Rayleigh, in which it appears that for a space with parallel walls, there is a triply infinite series of modes of vibration, the lowest mode in each series being that for which one of the three dimensions is a half wavelength of sound corresponding to that mode. In a beautiful series of experiments Knudsen[1] showed that for low tones in a small room $8\times8\times9.5$ feet, the reverberation is the free damped vibration of the entire volume of air in one or more of its natural modes, and that the picture of random moving waves does not describe the phenomena at all in a room as small as this for low pitched sounds.

Now our reverberation chambers are considerably larger than the small room of Knudsen's experiment. Nevertheless they are still so small that the picture of the free vibration of a three dimensional body of an elastic fluid more nearly represents the true conditions of the decay of sound energy than does the statistical picture of freely moving particles in a three dimensional space. In passing it may be noted that assuming a damping term in our differential equation which is a linear function of the particle velocity, the three dimensional vibration concept leads to a logarithmic decay of *average* intensity, but one which fluctuates widely about the logarithmic curve if measured at a single point. Now the number of possible modes of vibration within a small frequency range in a room whose dimensions are more than two or three wavelengths of sound of the driving frequency is very great. For example, computing from the Rayleigh formula,

$$n=\tfrac{1}{2}c[p^2/l^2+q^2/w^2+r^2/h^2]^{\frac{1}{2}},$$

we find that for a room $25\times30\times20$ feet, there are at least six different modes of vibration all lying within the frequency range 450 to 455 cycles. That is to say, in such a room any frequency above 250 cycles, let us say, would be very close to a number of natural frequencies of the room. Our reverberation chamber thus presents itself as a three dimensional flat response resonator with a large number of degrees of vibrational freedom within a small frequency range for any except very low frequencies. Damping takes place only at the bounding surfaces, and (neglecting atmospheric absorption) the rate of decay depends upon the average absorptivity of these surfaces. Excited by a source of any given frequency, presumably one single mode of vibration will be set up, that mode being determined by the boundary conditions, the source itself being a unique portion of the boundary. When the source is stopped, the boundary conditions are suddenly changed and the vibration continues in one or more of the possible natural modes of the room with frequencies close to the driving frequency.

Now if we assume that the energy is equally partitioned among all the possible modes of vibration, and we assume that all the bounding surfaces have equal absorptivities, it might be supposed that under these conditions the average decay should be strictly logarithmic. The dissipation of each particular mode will be so, but by inspection of the Rayleigh formula we note that there are a large number of possible values

[1] V. O. Knudsen, J. Acous. Soc. Am. **4**, 20 (1932).

of p, q and r that satisfy the equation for any given value of n. For example, suppose that the smallest dimension h of the room is an integral number of half wavelengths. Then one possible mode is $n=cr/2h$, p and q both being zero and the vibration in this mode takes place along this shortest dimension. The energy in this mode would be most quickly damped because the distance of travel between reflections is less than for other modes. In other words, looked at from this point of view, there is a tendency as the decay progresses for an increasing proportion of the energy to reside in the modes corresponding to the longest time of travel between reflections and a consequent decrease in the average rate of dissipation. In other words, even with uniformly distributed absorption, the decay is not strictly logarithmic.

It is apparent that neither a moving source nor a moving pick-up would be effective in neutralizing this effect. Reflecting surfaces moving during the decay process would tend to do it, however.

Consider now the effects from the three dimensional resonator point of view of introducing a small area of a highly absorptive medium into a room whose bounding surfaces are otherwise highly reflecting. Take an extreme case and assume that, save for the absorbent patch, the walls are perfectly reflecting. Then in the steady state, with the source still on, there will be at every point of the room a flow of energy toward the absorbent patch and in no other direction, since in every other direction the flow of energy toward a perfectly reflecting surface will be exactly equal to that away from it. This immediately vitiates the assumption in our definition that all directions of energy flow are equally probable. Examine the situation in detail by considering what happens at the edge of the absorbent material in the case of a wave normal to its surface. Let the wave, Fig. 1, originate at S and strike the surface NM, the left half of which is perfectly reflecting and the right half perfectly absorbing. The Huygens' construction for the reflected wave is shown with the diffraction fringe at the dividing line between the reflecting and absorbing portions of the surface. It is obvious that, because of diffraction, not only that portion of the incident

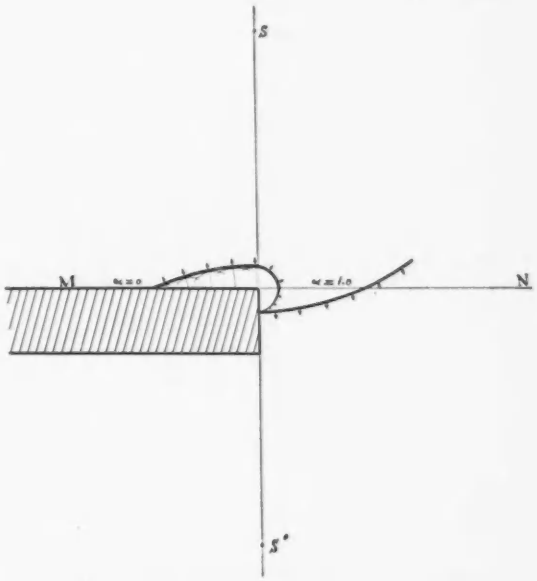

FIG. 1. Huygens' construction showing diffraction at the edge of a highly absorbing surface.

wave which strikes the absorbing surface is absorbed, but some energy is extracted from that portion of the wave front which is incident upon the adjacent reflecting portion of the surface as well. In other words, there will be in the steady state a flow of energy parallel to the reflecting surface toward the absorbing surface. This accounts for the so-called "edge effect" and explains the fact that with small areas of highly absorbent materials an apparent absorptivity greater than unity is frequently measured. Looked at in another way, if the root mean value of the sound pressure in the incident wave is p, then at the reflecting surface the total pressure is $2p$, and this increased pressure extends over a small portion at the edges of the absorbent surface. Similar reasoning leads to the general conclusion that with localized absorption, there will be at every point of the room, a net flow of energy toward the absorbent area, and that therefore no matter what we do in the way of using flutter tones, moving source, moving pickup, or rotating vanes, the assumption of a random direction of sound energy flux will not hold strictly.

When the source is stopped, the absorption being localized, this flow of energy from all points of the room toward the absorbent area

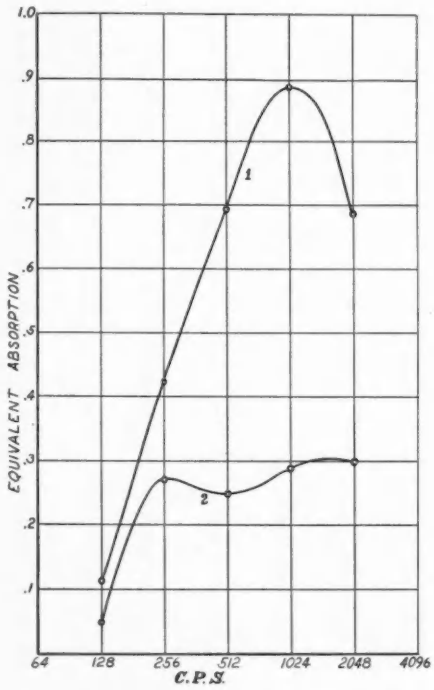

FIG. 2. Equivalent absorption per unit area. (1) 2.03 square meters in strips 25 cm wide arranged to form a hollow rectangle. (2) 1.95 square meters placed inside the hollow rectangle.

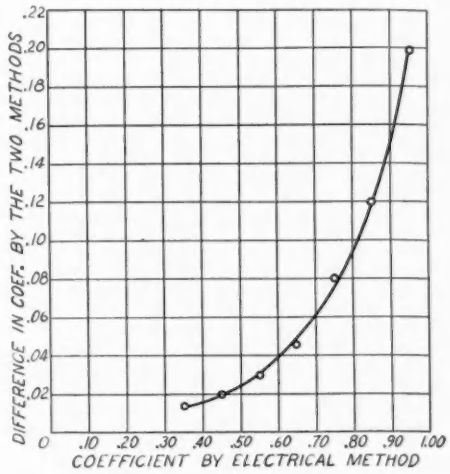

FIG. 3. Ordinates are differences between apparent absorptivity measured by ear method for 90–100 db decay and measured electrically for 40 db. Abscissae are electrically measured absorptivities.

persists with the result that the net energy flux incident upon the absorbing surface throughout the decay is greater than that toward an equal non-absorbent area. This disproportion is greater, the greater the intrinsic absorptivity and the smaller the area of the absorbent surface. We have already considered in detail diffraction at the edge of the sample. We see here the explanation of the high apparent absorptivities of small samples, and of the fact that a given area at the periphery shows a greater equivalent absorption than an equal area at the center of an absorbent sample (Fig. 2). Localized absorption tends to increase still further the departure from strict logarithmic linearity in the decay. While little is known for certain as to the variation of absorptivity with the angle of incidence, yet it is fairly safe to assume that it is greater at normal incidence than at grazing incidence. Consequently modes in which the direction of propagation is normal to the reflecting surface will be more rapidly absorbed than those which are parallel to it, with the result that the decibel level falls more rapidly in the initial than in the later stages. This is strikingly brought out by recent experiments of Chrisler's at the Bureau of Standards in which the entire floor of the reverberation chamber was covered with a highly absorbent material and the decay was measured with a directional microphone. With the ribbon placed vertically so as to respond to horizontal particle velocities, two effects were noted: (1) The measured initial intensity was higher, and (2) the rate of decay was somewhat less than when the ribbon was placed horizontally so as to respond to vertical particle velocities. In neither case was the decay logarithmic. These effects were not present when a low absorption material covered the reverberation chamber floor.

Chrisler also found that when the entire floor of the reverberation chamber is covered with a highly absorbent material, the decay is definitely not logarithmic even when measured with a non-directional microphone.

Bearing on this same point, is the fact that the absorption coefficients of materials measured on 72 sq. ft. samples by ear methods with a decay of from 85 to 100 decibels are lower than the values obtained by electrical measurements over 40 or 50 decibels (Fig. 3).

Both theory and experiment force us to recognize that the logarithmic law is only an approximation to the actual facts of reverberation in rooms whose dimensions are only a few

TABLE I.

Laboratory	Volume	Test area sq. ft.	COEFFICIENTS 128	256	512	1024	2048
Bureau of Standards	15,000	72	0.12	0.41	0.96	0.99	0.68
" " " (corrected values)			.12	.41	.90	.92	.67
National Physical Lab.	6,330	100		.33	.78	.86	.74
" " "	4,600	100		.34	.69	.84	.69
University of Illinois		72		.58	.82	.69	.82
Riverbank	10,000	72	.10	.42	.89	.95	.69
Electrical Research		71		.49	.80	.71	.52
Heinrich Hertz Institute			.10	.31	.69	.69	.59
Johns-Manville			.11	.41	.92	.87	.76

wavelengths of sound. Instead of diffuse distribution we have a distribution in which the flow of energy toward the absorbent sample is more probable than toward an equal non-absorbent area. What within the limits of experimental accuracy appears to be a straight line is but a segment of a curve of slight curvature. Using electrical methods, we are working over the steepest portion of the curve, where slight departures from linearity are masked by the fluctuating character of the reverberant intensity. With the organ pipe and ear method we get the two end points of a much greater portion of the curve, and take the slope of the chord as the average rate, and consequently get a lower value of the apparent absorptivity (Fig. 4).

If the picture which has been presented of the phenomenon of reverberation is correct, then it must follow that the coefficient of absorption of a material measured by present methods is a function not only of the intrinsic absorptivity of the material itself, but of the volume of the reverberation chamber, the absolute area of the test sample and the ratio of this area to the total surface of the room. This unfortunately seems to be the case. Table I shows the results obtained in different reverberation chambers, upon samples of Triple B Acousti-Celotex taken from the same factory run of material and supplied by the Celotex Company to the various laboratories for the purpose of these comparative tests. The figures are taken from the report of Professor Watson, Chairman of the Committee on sound absorption, under date of November 15, 1933.

Full details of the method and conditions of measurement are not available in all the cases but the methods at the Bureau of Standards, the National Physical Laboratory and at the Riverbank Laboratories were essentially the same, namely, measurement by electrical timing of the first 40 or 50 decibels of the decay (Table II). The National Physical Laboratory figures are for 100 square feet of material and the coefficients are computed by the formula,

$$T_0 = -0.05\,V/S \log e(1-\alpha).$$

Those at the Bureau of Standards and Riverbank

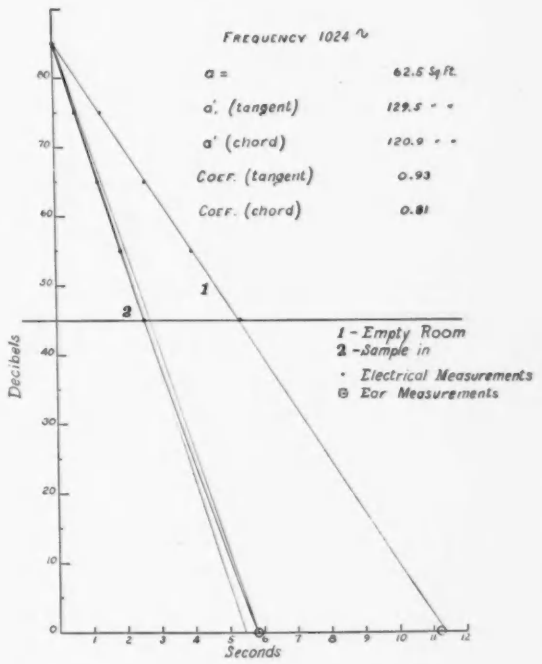

FIG. 4. Rates of decay measured electrically and by ear method.

by the formula,

$$T_0 = 0.05 V/a.$$

Use of the latter formula would raise the National Physical Laboratory values slightly.

A comparison of the results of measurements on the same material in four different rooms in three laboratories is instructive (Table II).

TABLE II.

Laboratory	Dimensions	Volume	Total floor area	Per-cent covered
1. Bureau of Standards	30x25x20	15,000	750	9.6
2. Riverbank	27x19.4x19	10,000	524	13.8
3. National Phys. Lab.	29.5x23.5x9.25	6,330	665	15.0
4. National Phys. Lab.	21x12.5x17.5	4,600	262	38.3

COEFFICIENTS

	128	256	512	1024	2048
1.	0.12	0.41	0.96	0.99	0.68
2.	0.10	0.42	0.89	0.95	0.69
3.		0.33	0.78	0.86	0.74
4.		0.34	0.69	0.84	0.69

It seems obvious from inspection of the figures in Table II that the absorption coefficient as determined by rate of decay measurement is a function of something beside the intrinsic absorptivity of the material. We note at 512 cycles a consistent decrease in the coefficient with decreasing volume of the room and increasing percentage of floor area covered. This is less marked at 1024, and at 2048 the measured value of the coefficient is independent of the room size. The explanation seems to be that all the rooms are *large* rooms measured in wavelengths for the frequency 2048 cycles, and the coefficient is a large area coefficient, while at 512 cycles the same absolute sample area is a *small* area for the 750 square feet of the Bureau of Standards total floor area, and a large area for the smaller rooms of the National Physical Laboratory.

A study of the curves showing the relation between sample area and apparent absorptivity given in Chrisler's very excellent paper on Dependence of Sound Absorption and Distribution of the Absorption Material[2] bears out the essential correctness of this point of view.

[2] Chrisler, Bureau of Standards Research Paper R.P. 700.

It would appear that all of the foregoing leads to the general conclusion that what we are measuring in a reverberation chamber is the effect per unit area of a particular area of the material on the rate of decay in that particular room. As to the general applicability of the values so obtained to the general problem of any area of the material in any room we are still very much in doubt. Fortunately the precision of the value assigned to the coefficient of any given material isn't very important either in the design of acoustically good rooms or in problems of quieting. The fact that the human ear has a roughly logarithmic intensity response takes care of that. It is only when we come to the commercial aspects of the matter, in the writing of specifications for acoustical installations and the sale of materials that the problem becomes important. Here it is important that testing laboratories should agree on a nominal value, even though that value is arbitrarily defined and arrived at by arbitrarily prescribed conditions.

Chrisler, in the paper already cited proposes to apply a correction to the coefficient obtained on 72-square foot samples. This correction is independent of frequency and is a function of the value of the coefficient obtained by measurements of a 72-square foot sample, and ranges in value from 0.09 for a measured value of 1.05 to 0 for a measured value of 0.60. This correction reduces the coefficients obtained on the 72-square foot area to values which would presumably be obtained on areas so large that diffraction effects are negligibly small. Applying these corrections to the Bureau of Standards figures on the comparison sample of Acousti-Celotex brings them down very close to the uncorrected values for 72 square feet obtained in the smaller Riverbank chamber but leaves them still considerably greater than the National Physical Laboratory values obtained in still smaller rooms. This leaves the universal application of these corrections to values obtained by rate of decay measurements on a fixed area in any room very much in doubt.

A suggestion that might be offered in our present dilemma as to how to arrive at some sort of agreement in our tests is this: Suppose tests are made in different rooms by identical methods, not on a fixed area of material, but

upon an area in each case which is a given percentage of the total surface of the room. Would we then get the same measured values for the equivalent absorption per square foot? The question is one which can only be answered by experiments. This is suggested to Professor Watson as a possible next step in our cooperative research on this question.

The proposal to abandon the reverberation method meets the obvious rejoinder that so far it is about the only method that tells us even roughly what we want to know. Various tube and box methods have been employed, but all are open to one of two objections, either that they do not give us reverberation coefficients, or that their use depends upon our having comparison materials of known reverberation coefficients.

Several years ago Knudsen[3] used a method based on the assumption, that with a source of fixed acoustic output, the intensity is inversely proportional to the total equivalent absorption. Total equivalent absorptions are thus compared by intensity measurements with and without the absorbent sample present. Recently Norris has used this method in a small room in which was mounted a rotating vane so large as barely to clear the walls. His results showed an increase of total equivalent absorption that was proportional to the area of the absorbent material for areas ranging from 2 to 16 square feet. Knudsen's results also showed no systematic variation of the equivalent absorption per square foot for areas ranging from 4 to 127 square feet.

It is possible that by some such means we can avoid the troublesome area effects. However, it is to be noted that the method presupposes that we know either the equivalent absorption of the empty chamber or the absorptivity of some standard material, and for this information we are forced to go back to reverberation methods of measurement.

Furthermore it is to be borne in mind that in using the intensity method we are assuming in lieu of any means of measuring total acoustic output of the source that a given power input to the source will produce a constant acoustic output when room conditions are altered by the introduction of absorption. This assumption is not strictly true in theory, and this fact may lead to as wide differences in the results of measurements made under different conditions in different laboratories as now exist in the results of reverberation measurements.

[3] Knudsen, Phil. Mag. **5**, 1240 (1928).

The Motion of a Bar Vibrating in Flexure, Including the Effects of Rotary and Lateral Inertia

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In this paper a complete theoretical solution is given for a bar vibrating in flexure taking account of rotary and lateral inertia. The solution shows that the frequency of a bar free to vibrate on both ends, is asymptotic to the frequency given by the usual solution, neglecting rotary inertia, when the ratio of width to length is small, and approaches the frequency of a bar in longitudinal vibration when the width becomes comparable to the length. The theoretical frequencies have been compared with the published results of Harrison on the frequency of a quartz crystal vibrating in flexure, and have been found to agree within one percent for a crystal whose width is less than half its length.

INTRODUCTION

IN studying the modes of motion of a quartz crystal, it has been found desirable to determine the frequency of a bar vibrating in flexure. The motion of such a bar has previously been solved when the rotary inertia has been neglected and Rayleigh[1] has given an approximate method for determining the correction due to the rotary inertia. However, when the width of the bar becomes comparable to the length, the experimental results obtained deviate considerably from these theoretical results.

Accordingly, this paper gives a complete solution for a bar vibrating in flexure taking account of the rotary inertia, and also the lateral inertia. These theoretical results have been compared with the measured frequencies of a quartz crystal vibrating in flexure published by Harrison,[2] and have been found to agree well with the experimental results.

SOLUTION FOR THE EQUATION OF MOTION OF A BAR VIBRATING IN FLEXURE

The equation of motion[1] for a bar vibrating in flexure is

$$\frac{\partial^2 y}{\partial t^2}+b^2k^2\frac{\partial^4 y}{\partial x^4}-k^2\frac{\partial^4 y}{\partial x^2\partial t^2}=0 \qquad (1)$$

where $y(x)$ is the displacement of the central line of the bar from its equilibrium position, at a distance x from one end of the bar, E = Young's modulus, ρ = density, b = velocity of sound $=(E/\rho)^{\frac{1}{2}}$, and k is the radius of gyration of the section. For a rectangular section $k=w/\sqrt{12}$ where w is the width of bar. The boundary conditions to satisfy for a bar free to move on the ends—which is the case of interest for a vibrating crystal—are[1] $\partial^2 y/\partial x^2=0$, when $x=0$ and $x=l$, the length of the bar and

$$b^2\partial^3 y/\partial x^3-\partial^3 y/\partial t^2\partial x=0 \quad \text{when } x=0 \text{ and } l. \qquad (2)$$

For simple harmonic motion, Eqs. (1) and (2) reduce to

$$-\omega^2 y+b^2k^2\partial^4 y/\partial x^4+\omega^2k^2\partial^2 y/\partial x^2=0, \qquad (3)$$

$$\frac{\partial^2 y}{\partial x^2}=b^2\frac{\partial^3 y}{\partial x^3}+\omega^2\frac{\partial y}{\partial x}=0 \quad \text{when } x=0 \text{ and } l. \qquad (4)$$

This equation takes account of the rotary inertia through the inclusion of the last term in (3) but takes no account of the lateral inertia caused by displacement of the body toward the center line due to the bending of the bar. A correction for this is discussed later. The only complete solution of (3) previously obtained neglects the last term containing the rotary inertia.

A solution for (3) is easily obtained by using the ordinary methods of solving partial differential equations by writing

$$y=[A\sin\alpha x+B\cos\alpha x+C\sin\beta x+D\cos\beta x], \qquad (5)$$

where A, B, C and D are arbitrary constants. Substituting this value in Eq. (3), we find that it is a solution provided α and β satisfy the equations

[1] Rayleigh, *Theory of Sound*, Vol. I, Chapter VIII.

[2] J. R. Harrison, *Piezo-Electric Resonance and Oscillatory Phenomena with Flexural Vibration in Quartz Plates*, I. R. E., Dec., 1927.

$$\alpha^4-\frac{\omega^2\alpha^2}{b^2}-\frac{\omega^2}{b^2k^2}=0; \quad \beta^4-\frac{\omega^2\beta^2}{b^2}-\frac{\omega^2}{b^2k^2}=0. \quad (6)$$

Hence we take

$$\alpha^2=\frac{\omega^2}{2b^2}+\left(\frac{\omega^4}{4b^4}+\frac{\omega^2}{b^2k^2}\right)^{\frac{1}{2}};$$

$$\beta^2=\frac{\omega^2}{2b^2}-\left(\frac{\omega^4}{4b^4}+\frac{\omega^2}{b^2k^2}\right)^{\frac{1}{2}}. \quad (7)$$

To introduce the boundary conditions of interest (4) we have

$$\partial^2y/\partial x^2=-[\alpha^2(A\sin\alpha x+B\cos\alpha x)+\beta^2(C\sin\beta x+D\cos\beta x)], \quad (8)$$

$$\left(\frac{\partial^3y}{\partial x^3}+\frac{\omega^2}{b^2}\frac{\partial y}{\partial x}\right)=\alpha\beta^2[A\cos\alpha x-B\sin\alpha x]+\alpha^2\beta[C\cos\beta x-D\sin\beta x]$$

upon differentiating Eq. (5) and collecting terms. If these equations are set equal to zero when $x=0$ and $x=l$, we have the four relations

$$\begin{gathered}B\alpha^2+D\beta^2=0,\\ \alpha^2[A\sin\alpha l+B\cos\alpha l]+\beta^2[C\sin\beta l+D\cos\beta l]=0\\ A\beta+C\alpha=0,\\ \beta[A\cos\alpha l-B\sin\alpha l]+\alpha[C\cos\beta l-D\sin\beta l]=0.\end{gathered} \quad (9)$$

Eliminating B, C and D from these equations, there results

$$A[1-\cos\alpha l\cos\beta l-((\alpha^6+\beta^6)/2\alpha^3\beta^3)\sin\alpha l\sin\beta l]=0. \quad (10)$$

Since A cannot be zero, the expression in brackets must be zero, and this relation specifies the values of frequency possible to a bar in flexure.

Two limiting cases are of interest. When ω^4/b^4 is small compared to ω^2/b^2k^2, $\alpha^2=\omega/bk$; $\beta^2=-\omega/bk$, and the frequency Eq. (10) reduces to the usual frequency equation for a bar in flexure obtained by neglecting the rotary inertia term—this equation being

$$1-\cos(\omega/bk)^{\frac{1}{2}}l\cosh(\omega/bk)^{\frac{1}{2}}l=0. \quad (11)$$

On the other hand, when ω^4/b^4 is large compared to ω^2/b^2k^2 as it will be when the bar is wide compared to its length, the frequency equation is given by

$$\sin(\omega l/b)=0 \quad \text{or} \quad f=b/2l, \quad (12)$$

which is the frequency of a bar in longitudinal vibration. Hence, as the width increases, the frequency instead of increasing in proportion to the width, as indicated by Eq. (11), increases more slowly and finally becomes asymptotic to the value given by (12). This is a limit to be expected from physical considerations, for if a frequency higher than this could be obtained, it would require a given strain to be propagated with a velocity greater than the velocity of sound.

For a rod of rectangular section k, the radius of gyration, is equal to $w/\sqrt{12}$. Introducing this value into Eq. (10) and defining a value m by the equation

$$m=(\omega/bk)^{\frac{1}{2}}l=(\sqrt{12}\omega l/bR)^{\frac{1}{2}}=(\sqrt{12}\omega l^2/bw)^{\frac{1}{2}}, \quad (13)$$

where R is the ratio of width to length, Eq. (10) can be written

$$0=1-\cos\left\{m\left[\left(1+\frac{m^4R^4}{(24)^2}\right)^{\frac{1}{2}}+\frac{m^2R^2}{24}\right]^{\frac{1}{2}}\right\}\cosh\left\{m\left[\left(1+\frac{m^4R^4}{(24)^2}\right)^{\frac{1}{2}}-\frac{m^2R^2}{24}\right]^{\frac{1}{2}}\right\}+\left(\frac{3m^2R^2}{24}+\frac{4m^6R^6}{(24)^3}\right)\sin\left\{m\left[\left(1+\frac{m^4R^4}{(24)^2}\right)^{\frac{1}{2}}+\frac{m^2R^2}{24}\right]^{\frac{1}{2}}\right\}\sinh\left\{m\left[\left(1+\frac{m^4R^4}{(24)^2}\right)^{\frac{1}{2}}-\frac{m^2R^2}{24}\right]^{\frac{1}{2}}\right\}. \quad (14)$$

From this equation the values of m can be calculated for different values of R. Fig. 1 shows these calculated values for the first and second modes of motion. The frequency of a bar of given length, width, and elastic constants can be calculated from these values of m from the formula

$$f=(m^2w(E/\rho)^{\frac{1}{2}})/2\pi(12)^{\frac{1}{2}}l^2. \quad (15)$$

Correction for Lateral Inertia

Some experimental values for the frequency of a perpendicularly cut quartz crystal vibrating in flexure have been published by J. R. Harrison.[2] These values for the first mode of motion are shown by the points on Fig. 2. For a crystal cut in this manner, the value of $E=7.85\times10^{11}$ and $\rho=2.65$. Hence, from the values of m given on Fig. 1, and Eq. (15), the theoretical frequency

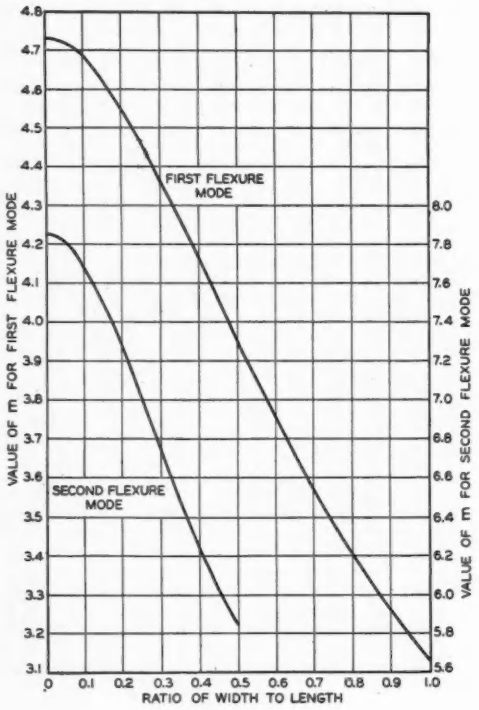
FIRST FLEXURE MODE
SECOND FLEXURE MODE
VALUE OF m FOR FIRST FLEXURE MODE
VALUE OF m FOR SECOND FLEXURE MODE
RATIO OF WIDTH TO LENGTH

Fig. 1.

can be calculated. It is shown by the dotted line of Fig. 2. As can be seen the curve deviates from a straight line in agreement with experiment but gives a frequency which is about 5 percent high when the ratio of width to length is 0.5. This divergence is due to the fact that other inertia corrections in other directions have not been taken account of. The most important of these is that due to the lateral contraction in the direction of motion, proportional to Poisson's ratio σ—due to the bending of the bar. The form of the differential equation containing a correction for this type of inertia has been investigated by Timoschenko,[3] who finds the equation to be

$$\frac{\partial^2 y}{\partial t^2}+b^2k^2\frac{\partial^4 y}{\partial x^4}-k^2(1+\sigma)\frac{\partial^4 y}{\partial x^2\partial t^2}=0, \qquad (16)$$

where σ is Poisson's ratio—counted as positive when a lateral contraction occurs for a longitudinal expansion. But this is the same form as Eq. (1) and hence the solution (5) holds for this case if

$$\alpha^2=\frac{(1+\sigma)\omega^2}{2b^2}+\left[\frac{\omega^4(1+\sigma)^2}{4b^4}+\frac{\omega^2}{b^2k^2}\right]^{\frac{1}{2}};$$
$$\beta^2=\frac{(1+\sigma)\omega^2}{2b^2}-\left[\frac{\omega^4(1+\sigma)^2}{4b^4}+\frac{\omega^2}{b^2k^2}\right]^{\frac{1}{2}}. \qquad (17)$$

The equation for determining the value of m is still (14) if R^2 is replaced by $R^2(1+\sigma)$. Hence, the value of m for a given value of R may still be obtained from Fig. 1 provided the abscissae are taken as $R(1+\sigma)^{\frac{1}{2}}$. For example if σ were 0.44 then the value of m for $R=0.2$ would be obtained from Fig. 1 by reading the value of m corresponding to $R=0.24$.

For a perpendicularly cut quartz crystal $\sigma=-s_{23}/s_{22}=0.121$ where s_{23} and s_{22} are two of the elastic constants of the crystal. Hence, with this value, the frequency can be calculated

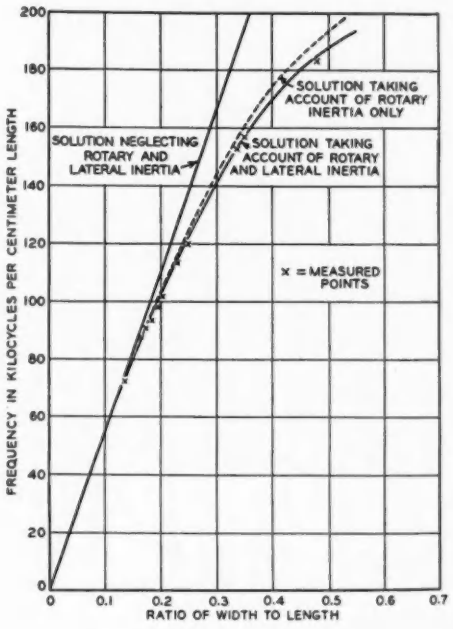
SOLUTION TAKING ACCOUNT OF ROTARY INERTIA ONLY
SOLUTION NEGLECTING ROTARY AND LATERAL INERTIA
SOLUTION TAKING ACCOUNT OF ROTARY AND LATERAL INERTIA
x = MEASURED POINTS
FREQUENCY IN KILOCYCLES PER CENTIMETER LENGTH
RATIO OF WIDTH TO LENGTH

Fig. 2.

[3] S. Timoschenko, Phil. Mag. (6), **41**, 744; **43**, 25; also Love's *Theory of Elasticity*, p. 430.

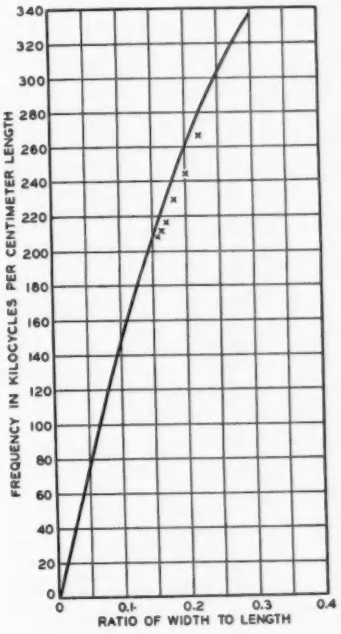

FIG. 3.

taking account of the lateral inertia in the plane of motion, and is shown by the solid line of Fig. 2. This curve agrees quite well with the measured points within about 1 percent. Since the inertia perpendicular to the plane of motion has been neglected it is probable that the remaining divergence is due to this. The correction for this appears to be quite small and since a differential equation of a higher order is required to account for this inertia correction, it is not considered in this paper. An approximate correction is easily obtained by Rayleigh's method.

The frequency for the second mode of flexure has been measured by Harrison for a few values of R. The measured points are shown on Fig. 3. By using the values of m given by Fig. 1 and the correction for lateral inertia, the theoretical values are shown by the solid line of Fig. 3. The theoretical values are about 5 percent higher than the experimental values. It is thought that the reason for this is the coupling of even order flexure modes of motion to shear modes of motion pointed out in a previous paper.[4] This coupling is not included in the ordinary theory of a bar vibrating in flexure.

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[4] *Electrical Wave Filters Employing Quartz Crystals as Elements*, Appendix, Bell. Sys. Tech. J. **13**, 405 (1934).

Performance of Telephone Receivers as Affected by the Ear

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An analysis is made of the impedance characteristic of the ear when coupled to a telephone receiver by means of an ear cap which introduces an acoustic leak at the junction. A vibrating system is also described which causes constant sound pressure to be generated in the ear cavity under such conditions. Experimental data as obtained on an artificial ear are given showing the effect of the acoustic leak on the response of various types of receivers. Subjective tests are also described and data given which corroborate the data obtained on the artificial ear.

INTRODUCTION

AN ideal telephone receiver should produce the same sound pressure at the tympanum of the ear as the original sound. In other words the introduction of the two electro-acoustic transducers, namely, the microphone and the telephone receiver, together with amplifiers and other associated reproducing equipment, should not change the response frequency characteristic of the sound pressure upon the tympanum as compared to the original sound.

When the head is immersed in a plane wave sound field, the pressure at the surface of the head at the ears is a function of the frequency of the sound and azimuth of the head with respect to the direction of propagation of the sound. The pressure upon a rigid spherical surface has been investigated theoretically by Stewart[1] and Ballantine.[2] Stewart has shown that these results can be applied to give the pressure at the surface of the head at the ears. These results show that below 1000 cycles the diffraction of the sound by the head is negligible. Furthermore, the dimensions and configuration of the ear cavity are such that the pressure at the bottom of the ear cavity is the same as that at the surface of the head. There is a variation in the ratio of the pressure at the ears to that in free space at the higher frequencies but not of sufficient magnitude to be significant. If the head is turned so that one ear faces the source of sound, the pressure at this point at the higher frequencies is twice that in free space, while the shadow cast by the head results in very small pressure upon the other ear. Of course the normal listening position is that of facing the source of sound. In rooms, in addition to the direct sound, there is also the generally reflected sound. If all directions of the reflected sound are assumed to be equally probable, there will be practically no frequency discrimination as regards any point on the surface of the head. The ratio of the pressure at the bottom of the ear cavity to the pressure at the surface of the head varies considerably from unity at the resonance frequency of the cavity. Of course, this resonance is modified by the introduction of the receivers. However, again the change of this resonance is perhaps not of great importance. The relative importance of the factors discussed above will be apparent in the subjective tests. Assuming that the ratio of the acoustic input to the electrical output of the microphone and amplifier are independent of the frequency, the ratio of the voltage applied to the telephone receivers to the resulting pressure in the ear cavity should be independent of the frequency.

The high quality ribbon type telephone receivers[3] have a specially shaped ear cap of soft rubber. When carefully placed upon the ears to avoid leakage the acoustic impedance presented to the receiver by the resulting cavity is practically a pure acoustic capacitance. In order that the ratio of the sound pressure in the cavity to the applied voltage shall be independent of the frequency the ratio of the amplitude to the applied voltage must also be independent of the

[1] G. W. Stewart, Phys. Rev. **14**, 376 (1919).

[2] S. Ballantine, J. Acous. Soc. Am. **3**, 319 (1932).

[3] H. F. Olson and F. Massa, *A High Quality Ribbon Receiver*, Proc. I.R.E. **21**, 673 (1933).

Olson and Massa, *Applied Acoustics*, P. Blakiston's Son and Company, Philadelphia, p. 149.

frequency. A system has been described which accomplished this objective and the response of the receivers when carefully placed upon the ears is essentially flat from 30 to 10,000 cycles.

In the case of telephone receivers with hard ear caps a leak occurs between the ear and cap. The acoustic impedance presented to the receivers is considerably more complex than in the case of the perfect seal. It is the purpose of this paper to describe a type of vibrating system which delivers constant sound pressure to the ear cavity in which a leak occurs between the cap and ear.

Theoretical Considerations

In order to design the vibrating system so that constant sound pressure will be delivered to the ear cavity for constant applied voltage, we must know the acoustic impedance looking through the aperture in the ear cap. This acoustic impedance has three components; namely, the resistance and inertive reactance due to the leak between the cap and the ear, and the capacitive reactance due to the cavity. Investigations of the impedance looking into the aperture of a receiver cap have been made by Inglis, Gray and Jenkins.[4] The equivalent electrical circuit of the ear cap and ear cavity, together with graphs showing the resistive and reactive components are shown in Fig. 1. An examination shows that the impedance is positive and increases with the frequency up to 400 cycles—between 300 and 500 it is practically resistive—and above 400 it is negative and decreases with frequency. To maintain constant sound pressure in the ear cavity, with the above conditions, the velocity of the diaphragm below 300 cycles must be inversely proportional to the frequency, between 300 and 500 cycles the velocity should be independent of the frequency, and above 500 cycles the velocity should be proportional to the frequency. This is a generalization of the requirements.

A system is shown in Fig. 2 which, with properly chosen constants, delivers practically constant sound pressure to the ear cavity in the range from 50 to 7000 cycles. This system

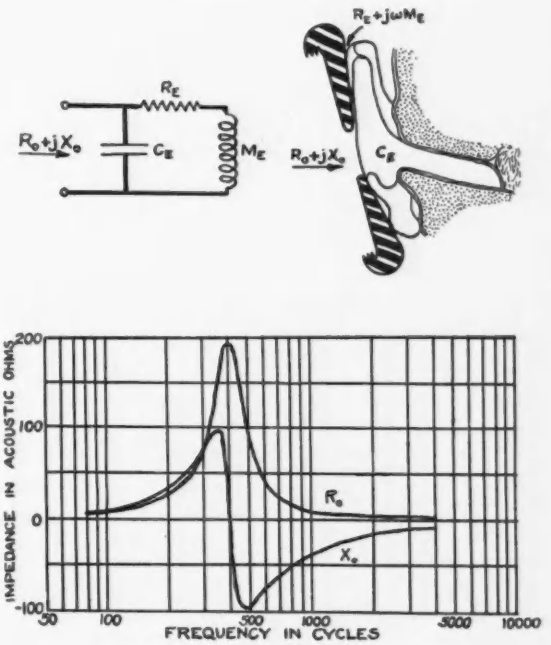

Fig. 1. Acoustic impedance of the human ear as viewed through the ear cap of a telephone receiver. (After Inglis, Gray and Jenkins.)

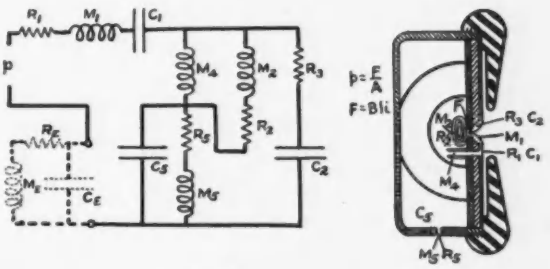

Fig. 2. Construction of the receiver showing the vibrating system and equivalent electrical circuit to deliver constant sound pressure to the ear cavity in the presence of a leak between the ear cap and the ear.

consists of a V-shaped diaphragm M_1 driven by a straight conductor located in the bottom of the V, a suspension system R_1C_1 supporting the diaphragm and aligning the conductor, a permanent magnet field structure, a cavity back of the diaphragm R_3C_2, a bolt of cloth forming an acoustic resistance and reactance R_2M_2, a case C_5 and a hole in the case M_5R_5. The equivalent electrical circuit of the acoustical system coupled to the equivalent electrical circuit of the ear is shown in Fig. 2. The acoustic impedance at the point p, Fig. 2, assuming the impedance of the ear to be zero, is:

[4] Inglis, Gray and Jenkins, Bell Sys. Tech. J. **11**, 293 (1932).

$$Z_T = Z_1 + \frac{[Z_5 + Z_2Z_4/(Z_2+Z_4)]Z_3}{Z_3 + Z_5 + Z_2Z_4/(Z_2+Z_4)} \qquad (1)$$

where $Z_1 = R_1 + j\omega M_1 + 1/j\omega C_1$,
$Z_2 = R_2 + j\omega M_2$,
$Z_3 = R_3 + 1/j\omega C_2$,
$Z_4 = j\omega M_4$,
$Z_5 = \dfrac{(R_5 + j\omega M_5)/j\omega C_5}{R_5 + j\omega M_5 + 1/j\omega C_5}$,

R_1 = acoustic resistance of suspension;
M_1 = inertance of diaphragm;
C_1 = capacitance of the diaphragm suspension;
R_2 = acoustic resistance of the bolt of silk back of the diaphragm;
M_2 = inertance of the bolt of silk;
C_2 = capacitance of the cavity back of the diaphragm;
R_3 = acoustic resistance of the cavity back of the diaphragm;
M_4 = inertance of the tube connecting the cavity back of the diaphragm and case;
R_5 = acoustic resistance of the hole in the case;
M_5 = inertance of the hole in the case;
C_5 = capacitance of the volume of the case.

The quantity p, the pressure in bars, shown in Fig. 2 is given by the expression,

$$p = Bli/A \qquad (2)$$

where B = flux density in the air gap in gauss;
l = length of the conductor in centimeters;
i = current in abamperes; and
A = effective area of the diaphragm in sq. cm.

The pressure in the ear cavity C_E (Fig. 1) in terms of the pressure p, is

$$P_E = \frac{p(R_E + j\omega M_E)/j\omega C_E}{Z_T/j\omega C_E + Z_T(R_E + j\omega M_E) + (R_E + j\omega M_E)/j\omega C_E}, \qquad (3)$$

where R_E = resistance of the leak between the ear and receiver cap;
M_E = inertance of the leak;
C_E = capacitance of the ear cavity; and
Z_T = total acoustic impedance of the receiver as given by Eq. (1).

The theoretically predicted response from Eq. (3), assuming constant current in the conductor, is shown in Fig. 3. This curve indicates reasonably uniform response from 50 to 7000 cycles.

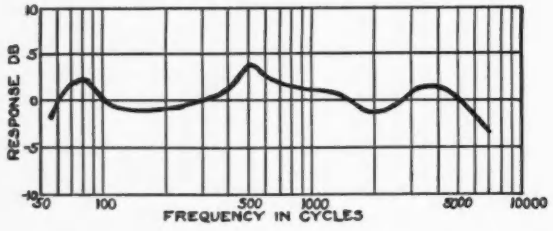

FIG. 3. Theoretically predicted response of the receivers in conjunction with the human ear. Computed from the circuit and constants of Fig. 2.

EXPERIMENTALLY DETERMINED RESPONSE CHARACTERISTICS

A detailed discussion on telephone receiver testing has been given elsewhere.[5] It will suffice at present to say that the frequency response characteristic of a receiver should indicate the pressure developed in an average ear at various frequencies for constant applied voltage to the receiver. One of the variables to be considered when testing receivers is the normal leak that results at the ear cap when they are placed on the head. The effect of this leak was indicated in Fig. 1 which shows the averages of a set of data taken on human ears with hard receiver caps worn in the conventional manner.

(a) Artificial ear tests

An artificial ear was built up as shown in Fig. 4, and consists of a condenser microphone in which is mounted an adapter to reduce the cavity to about 4 cc. A series of slits is placed between the microphone and receiver, introducing the inertance M_E and the resistance R_E as shown in the equivalent diagram. The impedance characteristic when looking into the receiver cap is also shown in Fig. 4 and is comparable to the set of curves shown in Fig. 1. By mounting the receiver under test as shown in Fig. 4 and supplying the unit with constant voltage, the pressure developed in the cavity is taken as the response at the various frequencies.

The solid line in Fig. 5 shows the frequency-response characteristic of the new receivers taken

[5] Olson and Massa, *Applied Acoustics*, P. Blakiston's Son and Company, Philadelphia, Chapter IX.

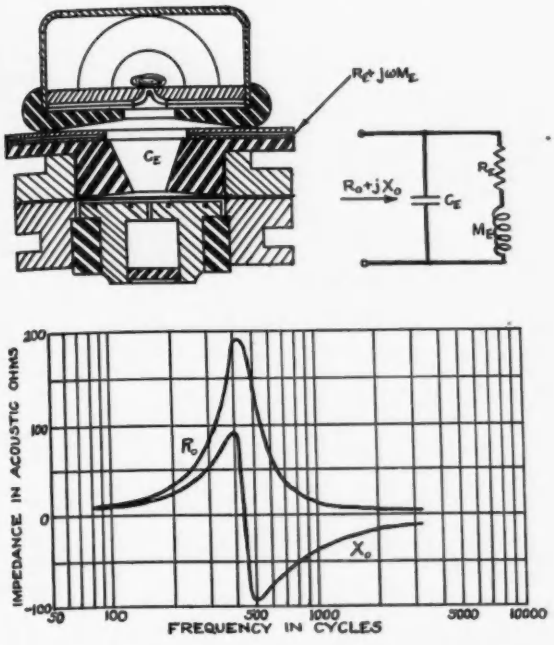

FIG. 4. Artificial ear and its equivalent electrical circuit. The curves show the acoustic impedance characteristics of the components of the system.

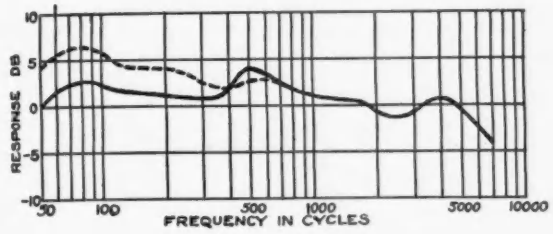

FIG. 5. Frequency-response characteristic of the receivers taken on the artificial ear. Dotted curve shows response with a perfect seal ($M_E R_E = \infty$ in Fig. 4). 0 db = 1 bar. Power input = 0.16 milliwatt.

on the artificial ear. The dotted curve shows the small change in response resulting from removing the leak at the ear and indicates that the variation in response with various leaks is very small.

In Fig. 6, the solid curve represents the response characteristic of the conventional type of bi-polar receiver taken on the artificial ear. The dotted curve shows the increase in low frequencies obtained by removing the leak. These two curves indicate that in this type of receiver the low frequency sensitivity becomes a function of how well the units are fixed to the ears.

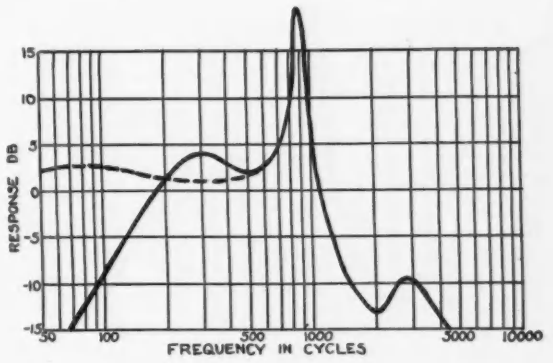

FIG. 6. Frequency-response characteristic of a conventional bi-polar type receiver taken on the artificial ear. Solid curve taken with normal leak. Dotted curve taken without leak ($M_E R_E$ in Fig. 4 = ∞). 0 db = 1 bar. Power input = 0.16 milliwatt.

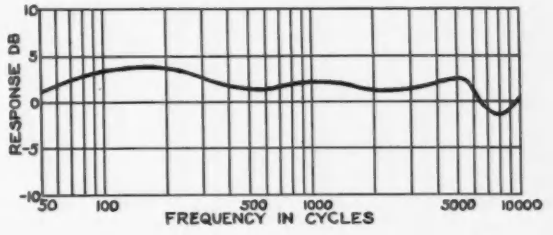

FIG. 7. Frequency response characteristic of the ribbon type receiver taken on the artificial ear without leak. 0 db = 1 bar, power input = 1.0 milliwatt.

In the case of the ribbon receivers a special soft rubber ear cap is used and when carefully placed on the ears no leak results. The response on the ribbon telephone receiver without a leak is shown in Fig. 7.

(b) Subjective tests

To confirm the characteristics as obtained on the artificial ear, subjective measurements were made while the receivers were actually worn by various individuals. The schematic arrangement for this test is shown in Fig. 8. A frequency is impressed on the loudspeaker and the oscillator output is adjusted until the microphone amplifier output meter reads a predetermined value. With the receivers removed, the observer listens to the intensity of the sound, then he places the receivers on his head, switches the oscillator output across the potentiometer and adjusts the voltage across the receivers until the intensity in his ears is the same as the free wave intensity from the loudspeaker. This procedure is repeated for several frequencies keeping the free wave

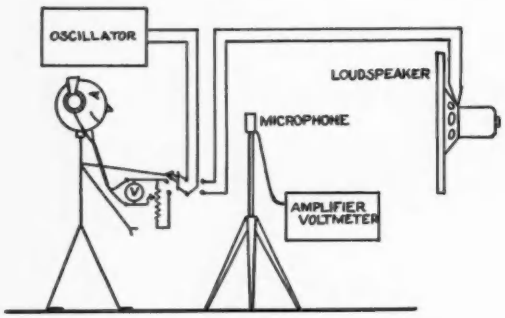

FIG. 8. Schematic arrangement for subjective tests on telephone receivers.

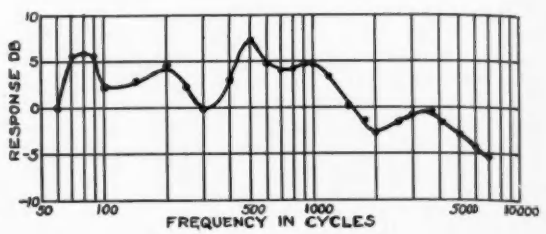

FIG. 9. Frequency-response characteristic of the receivers. Response obtained by comparing with constant sound pressure in free space (see Fig. 8). Plotted points are averages of six observers. 0 db = 1 bar. Power input = 0.16 milliwatt.

pressure constant, and the reciprocals of the voltages across the phones required to match the free wave sound intensity are proportional to the sensitivity of the receivers at the various frequencies.

Six individuals were independently used for this test and the average frequency response characteristic as determined from these measurements is shown in Fig. 9. The maximum deviation from this mean curve for any individual was less than 3 db.

Fig. 9 checks closely enough with the characteristic shown in Fig. 5 to justify the value of the acoustic leak that was used in the artificial ear to simulate the normal ear leakage at the receiver cap. The further agreement of Fig. 9 with the theoretically predicted response shown in Fig. 3 substantiates the analysis.

Quantitative Studies on the Singing Voice

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INTRODUCTION

HERETOFORE exact consideration of the art of singing has been handicapped by the lack of suitable quantitative means for simply evaluating voice characteristics. Recently developed acoustical devices have made possible the inauguration of a series of physical tests on the singing voice resulting in a large amount of objective data of considerable potential significance. The factors studied include intensity as a function of time and pitch, vibrato, vibrato-tremolo and tremolo and quality. The fundamental aim of our work is to evaluate the caliber of a voice as completely as possible insofar as its external physical manifestations are concerned. Ultimately, we hope that it will be possible to establish a definite correspondence between these external manifestations and a singer's vocal equipment, thus obtaining an objective method for rating the latter. On the basis of present information it is possible to evaluate the performance of a singer in regard to some characteristics of his voice, and to demonstrate improvement or deterioration. It has also been found possible to evaluate certain phases of artistry depending on physical factors and mechanical control. We are, at this time, presenting a preliminary report of our findings.

DESCRIPTION OF INSTRUMENTS

The instruments used included an automatic level recorder,[1] a crystal analyzer[2] and an acoustic spectrometer,[3] all developed at the Bell Telephone Laboratories. As its name suggests, the automatic level recorder is a device for automatically securing a record of fluctuating sound pressure levels in decibels as a function of time. The frequency range extends from 30 to 10,000 cycles. The sensitivity of the stylus may be adjusted to follow intensity level changes up to 360 db per second. By driving the recorder paper and crystal analyzer tuning control synchronously, the horizontal axis may be made frequency instead of time, and a quick harmonic analysis obtained. The crystal harmonic analyzer has two separate filters incorporated, one giving a band width of 20 cycles, and the other of 200 cycles. The frequency range extends from 40 to 10,900 cycles. The acoustic spectrometer employs 144 reeds, each responding to a different frequency. The lowest reed frequency is 50 cycles, the other reeds being tuned at progressively increasing frequencies with a 2.93 percent interval; i.e., there are 24 reeds per octave. The maximum frequency is 3109 cycles. The reeds are adjusted to give equal response for equal input voltages, the deflections being proportional to the voltages and therefore to sound pressures at the pick-up microphone. A photographic attachment permits records to be obtained.

For all of our measurements the singers were located in an acoustically dead room with the instruments in an associated laboratory. A loud-speaking communication system between the two rooms permitted the transmission of instructions and the singing pitch. Normally, one experimenter remained with the singer for aural check and coaching, while another operated the electrical equipment. The use of a dead room with close microphone pick-up minimized the possibility of errors due to reflection interference. A Western Electric moving coil microphone was always employed as the sound pick-up.

PITCH INTENSITY MEASUREMENTS

If we consider ordinary physical experience, one type of measurement we would make on a singing voice is an examination of the acoustic output as a function of the singing pitch. In rating mechanical or electrical equipment, one criterion is the efficiency of the equipment over a given range of working conditions. In our case we have plotted the power as a function of the pitch range. While the ability to make a loud sound on a few tones does not indicate a good or well-used voice,

[1] E. C. Wente, E. H. Bedell and K. D. Swartzel, J. Acous. Soc. Am. **6**, 3 (1935).
[2] W. P. Mason, Bell Sys. Tech. J. July, 1934.
[3] C. N. Hickman, J. Acous. Soc. Am. **6**, 2 (1934).

the ability to sustain a high level of intensity over a wide range does indicate that the voice is being produced at high efficiency. In any case, one criterion for evaluating a voice is the power output available.

With this in mind, pitch-intensity measurements were made for more than fifty voices of various calibers, ranging from untrained singers and beginners to successful professional artists. The voices tested included various types of baritones, tenors, contraltos and sopranos. Unfortunately, no true, fully developed bass voice was available.

For such measurements, we used our automatic level recorder. The singer was placed in the acoustically dead room at a distance of fifteen inches from the pick-up microphone. He was told to sing the vowel "ah" at full intensity and as freely as possible. He was asked to start with the lowest tone which he could produce and to hold each tone for about five seconds. Readings were taken at whole tone intervals until the highest tone which he could produce was reached. The microphone was orientated on a level with the mouth and in such a direction as to secure sound incidence at an angle of thirty degrees from the normal. The microphone and level recorder were calibrated daily by means of a standard sound source.

In reading intensity values, the highest sustained part of the tracing was considered and where vibrato appeared, the repeated maxima were read. In most cases two sets of readings were made and the higher value for each tone plotted.

In the cases of women whose registers were isolated, but not coordinated, separate readings of the falsetto and the lower registers were taken so that, over a certain area of the range, two points were obtained. Where the registers were not isolated, only one curve could be obtained. A few readings were made of men's falsettos, but this test was not taken in most cases, since men should not use the falsetto in performance.

Figures given in this paper are all total sound output levels relative to one microwatt. They were computed from the measured intensity levels assuming hemispheric radiation from a singer's mouth. The assumption of hemispheric radiation seems the most likely one to make. However, a few tests we made indicated that the level existing fifteen inches from the side of the head were in the order of 4 db lower than those existing fifteen inches in front of the mouth. This difference varied with the singing pitch. With the singer facing away from the microphone and his lips fifteen inches from it, the intensity was down an additional 2 db. For vowel sounds other than "ah," the differences were of varying magnitudes. In general, the darker vowels showed smaller differences.

"Ah" was selected as the standard vowel to be sung, primarily because it is an open vowel and widely used in exercising. In order to investigate the relative intensities of the different vowels with regard to pitch, we took measurements over the ranges of five good baritones singing on the vowels "ah," "ay," "oo" and "ee." We then plotted the mean of these readings, as shown in Fig. 1. It will be noted that "ah" and "ay" are at

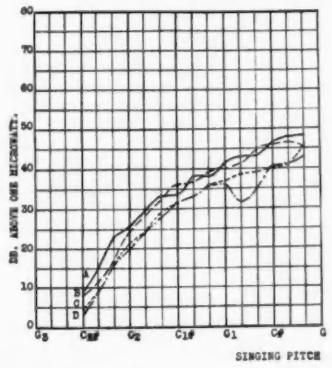

FIG. 1. Full voice power level *vs.* singing pitch. Average levels produced by five baritones. *A*. AH ————. *B*. AY — — — —. *C*. OO - - - - -. *D*. EE —·—.

a somewhat higher level than "oo" and "ee." This is not surprising in view of the fact that the mouth is generally in a somewhat closed position for the latter and relatively open for the former. The relatively smooth rise in intensity with pitch of three of these curves is an outstanding characteristic. For the vowel "ee" there is a sudden drop centering at the pitch of A below middle C (C = 261.6 cycles). This effect was extremely pronounced for some of the singers, but repeated measurements of the same voices did not show it to be consistently present. It is accompanied by a noticeable change in the quality of the vowel and the original quality being sung determines whether or not the drop occurs. The effect was also noticed for a good contralto and apparently

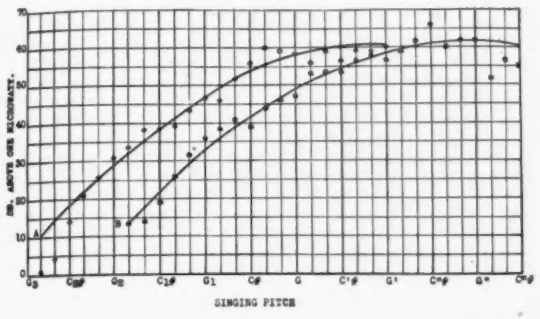

FIG. 2. Full voice power level *vs.* singing pitch. Reference curves. *A*. Men. *B*. Women.

is connected in some way with the vowel determinants.

Our subsequent measurements are all concerned with the vowel "ah."

In order to obtain a basis of comparison, it was decided to draw two reference curves, one for male and one for female voices. For these curves the points used corresponded to the highest levels obtained by any singer on the various tones. In drawing the curve for men's voices, allowance was made at the extreme low end for the fact that no real bass voice was available. The curves, as well as the determining points, are shown in Fig. 2.

The ranges considered were from A_3 to G′ (55 to 784 cycles) for men's voices and from A_2 to C‴ sharp (110 to 2216 cycles) for women's voices.

It will be seen that there is a uniform rise in intensity in both curves as the pitch rises, except at the extreme upper end. The extreme high points are not well determined since, in the women's curve, only one girl tested was able to sing above G″ and, in the men's curve, only one man was able to sing above D′ sharp. This may account for the droop at the top of both curves. For the balance of the curves, various singers were able to contribute.

It is possible that, if enough readings were taken from the voices of really great singers, the men's and women's curves would be exactly similar but an octave apart.

Various hypotheses are possible in order to explain the increase of intensity with pitch. As the pitch rises the tension on the vocal cords and on the muscles of the thorax increases. The resultant increase in breath pressure should partly account for the rise in intensity. Another possible explanation of increased intensity lies in the fact that, as the pitch rises, the lower harmonics generated by the larynx more nearly coincide with the resonance frequencies of the vowel sounds. If the assumption made by various people[4] that the distribution of energy among the partials generated by the vocal cords varies inversely as the cube of the harmonic number, then, when the lower partials of a high tone are accentuated through resonance, a higher level of energy should result. Still another consideration is that, if the amplitude of vibration remains constant with frequency, the energy should increase according to the square of the frequency. It is possible that all three factors enter into the phenomenon in some proportion. In any case, it is well known that a great deal more exertion is involved in singing a high tone than in producing a lower one. The phenomenon is in accordance with the common observation that, when one speaks loudly, he raises the pitch.

The maximum values measured for both men and women were of the same order of magnitude, i.e., one watt. This compares with the value of forty-one microwatts given by Fletcher[5] for the average phonetic power of the vowel in the word "talk." For extremely loud speech, Fletcher gives one thousand microwatts. The values we found do not appear excessive when the fact that a soloist can be heard over a large orchestra and a chorus is remembered.

As already stated, the reference curves are a composite of all types of voices. We find that good voices of lower tessitura (contralto and baritone) meet or cross the reference curves for the lower and middle tones while those of higher tessitura (soprano and tenor) meet or cross them for the upper tones and tend to fall below at the middle and, more especially, at the low ends. Of course, this is only true where the voice is properly developed.

EXAMPLES OF PITCH–INTENSITY READINGS

We will now consider some "pitch-intensity" curves taken on the voices of individual singers. In order to demonstrate the relative consistency

[4] J. C. Steinberg, J. Acous. Soc. Am. **6**, 1 (1934).
[5] H. Fletcher, *Speech and Hearing*, pp. 67, 68.

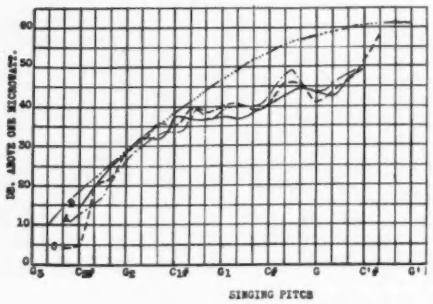

FIG. 3. Full voice power level *vs.* singing pitch. Stabilized baritone. *A*. 6/13/34 —·—. *B*. 7/25/34 ————. *C*. 9/5/34 - - - - -. Reference —·····—.

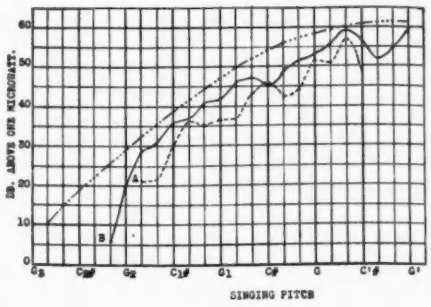

FIG. 5. Full voice power level *vs.* singing pitch. Advanced student tenor. *A*. 5/23/34 - - - - -. *B*. 9/26/34 ————. Reference —·····—.

of our results, there are shown in Fig. 3 three tracings taken at different times of a stabilized baritone voice. The curve taken on July 25, 1934 is lower throughout the upper range than are the other curves. On this occasion, the singer had a bad cold. Despite this fact, throughout almost the entire range the intensity variation is generally less than five db. Notice that the range is very wide, over three octaves, and that above $D_1\#$ the average increase of intensity with pitch is small relative to the reference curve.

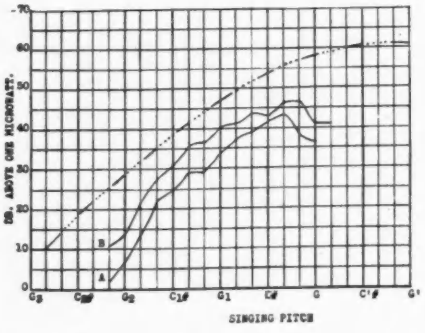

FIG. 4. Full voice power level *vs.* singing pitch. Beginner baritone. *A*. 6/7/34. *B*. 10/17/34. Reference —·····—.

Fig. 4 shows two curves for a young baritone in the beginner's stage. Curve *B* was taken four months later than *A*. The improvement in his voice is apparent since curve *B* is situated from five to eight db higher than curve *A* over most of the range. Note in curve *A* the sudden drop which starts at *D* sharp. In curve *B* this effect does not take place until the tone *F* is reached. Notice also that the range has been extended by one whole tone. The voice is of limited range and the power level is rather low. Curve *A* is over 20 db below the reference curve at the top and bottom of the range. The middle of the range is somewhat better, but even here it is about 13 db down. In curve *B* the intensity has risen, and in the middle of the range, comes within about 8 db of the reference curve.

The drops starting at D# and F, i.e., at the uppermost tones, are paralleled by similar drops at diverse pitches for other voices. Pronounced drops of this sort seem to indicate that the higher tones are poorly produced. One physiological explanation we may offer is that a constriction of the throat occurs at such points. Whatever it is due to, the condition apparently is ameliorated in the course of training.

The two curves in Fig. 5 are for a fine tenor voice under training. Curve *B* was taken four months later than *A* and shows not only a striking increase in power but also a wide increase in range at both ends.

Curve *A* is somewhat irregular, and through the middle of the range is often ten or more db below the reference curve. Curve *B* is more regular and comes up throughout the range to within 5 db of the reference curve except for the lowest tones. The slight drop of 2 db between *B* and C′ sharp in curve *B* may indicate a somewhat faulty adjustment for the C′# but the marked drop at D′#, which does not occur for the contralto of Fig. 6, may be taken to indicate some physiological effect; i.e., improper resonation, reduced laryngeal efficiency, or both. Except for this drop and the minor one at C#, curve *B* has a relatively smooth rise, which we might assume to be a desirable attribute.

That the young man has exceptional talent may be assumed because of his high power level over a wide range, uniform rise of intensity with

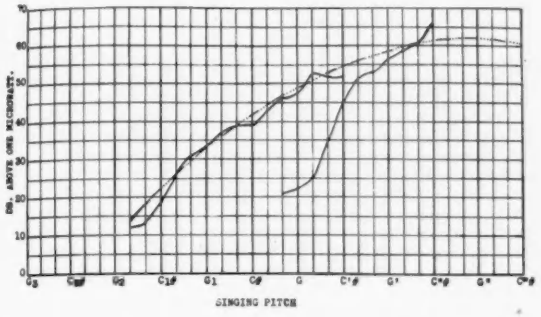

FIG. 6. Full voice power level *vs.* singing pitch. Advanced student contralto. Reference —·····—.

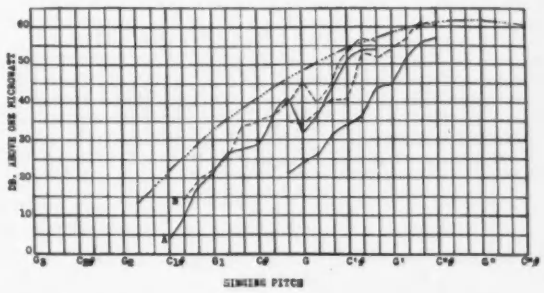

FIG. 7. Full voice power level *vs.* singing pitch. Soprano (See Text). *A*. 6/13/34 ———. *B*. 9/12/34 – – – –. Reference —·····—.

pitch, and from his susceptibility to training, judging by the progress shown by curve *B*.

Fig. 6 shows the curves for a fine contralto voice. In this case the lower register and the falsetto are plotted separately. Since equal intensities are shown on two consecutive tones; i.e., C′♯ and D′♯, the former tone being produced with lower register adjustment and the latter with falsetto, she is able to sing up the scale without any sharp transition in intensity. The point at which she should change from lower register to falsetto at full intensity is, therefore, D′♯. The curve crosses the reference curve at several points throughout the range with no material drop at any point and with the highest tone in the voice considerably above the reference curve. An unusually good voice of great power which should be effective throughout its extremely wide range is indicated. We have found with women's voices, that, where the range is wide and the intensity high over the range, the lower register is well developed.

Fig. 7 shows the "pitch-intensity" curves of a young soprano. These tracings are of particular interest because this girl, whose voice had been injured, had developed nodules on her vocal cords. At the inception of study, she was unable to sustain any tone and even her speaking voice broke. Before this time only the falsetto register had been employed for both the singing and the speaking voice and it had apparently been strained. The first curve was made some time after corrective treatment had been started. At this stage the lower register was well developed but the falsetto was still weak and the nodules on the vocal cords had not entirely disappeared. The second reading was made three months later when the nodules had disappeared and the strain on the falsetto mechanism was considerably alleviated. The interest in these curves lies principally in the wide change in the intensity of the falsetto throughout the range. The two registers were plotted separately for each occasion. The *A* curves are so far apart that there is obviously no point at which she could change register without an abrupt drop in intensity. The *B* curves show a smoothed out lower register of somewhat higher intensity, especially at *G*; while the falsetto shows an increase in intensity of up to 16 db. In this latter reading it may be seen that, if she changes register at *B*, there is no drop in intensity. The curves show a high intensity in the lower register at about C′ sharp and in the upper part of the falsetto which touches the reference curve at B′. The range covered is three octaves, although as yet the intensity does not rise uniformly.

Fig. 8 shows the "pitch-intensity" curves of two contraltos. Curve *A* is that of the student shown in Fig. 6 and curve *B* is for the voice of an opera singer of some standing.

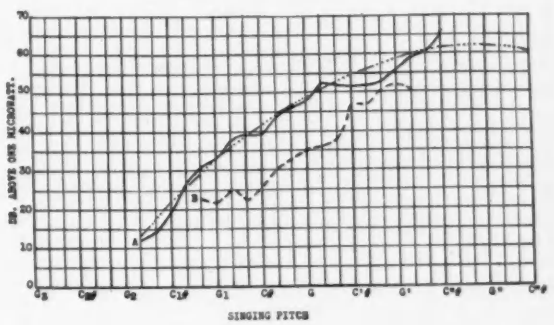

FIG. 8. Full voice power level *vs.* singing pitch. Two contraltos. *A*. Student ———. *B*. Opera performer – – – – –. Reference —·····—.

Considering curve *B* we find an intensity about 15 db below the reference curve throughout the middle of the range, and about 10 db below for the low tones. For the medium high tones (F′ and G′) the intensity is down less than 10 db At A′, the highest tone which this singer can reach, the intensity is down 10 db. The upper tones are lacking. Notice that the range covered by the opera singer is six whole tones less than that covered by the amateur. It was also observed that the former lacked lower register development.

It may be concluded from this and other similar cases we have tested that some singers are successful in spite of poor vocal equipment by reason of exceptional musical, dramatic or artistic attainments. It seems reasonable to assume, however, that such singers would have attained greater success with better vocal equipment.

Consideration of Vibrato Curves

Another factor of great importance in singing is the movement of the voice, which imparts a live, pulsating character to sung tones. Some sort of movement is essential not only from a listener's viewpoint, but when correctly effected, is considered to be technically advantageous.

A consideration of the physiological mechanism involved leads to the conclusion that voice movements of quite different characteristics may occur. We are arbitrarily classifying them as vibrato, tremolo-vibrato, and tremolo. We have reason to believe that, for what we define as vibrato, there occurs a movement of the thorax as well as of the laryngeal and pharyngeal muscles, whereas in the other cases there is no such thorax action. The vibrato is of lower frequency, ranging from about 4.5 to 6.4 per second, whereas the tremolo and tremolo-vibrato range from about 6.5 to above 8 per second. The distinction between the latter is one of degree. Where there is considerable variation in pitch and intensity at the higher range of speeds we term the movement tremolo-vibrato; otherwise we term it tremolo.

The intensity variation accompanying movement of the voice we measure with the automatic level recorder. As was to be expected, the variations differ between individuals and between

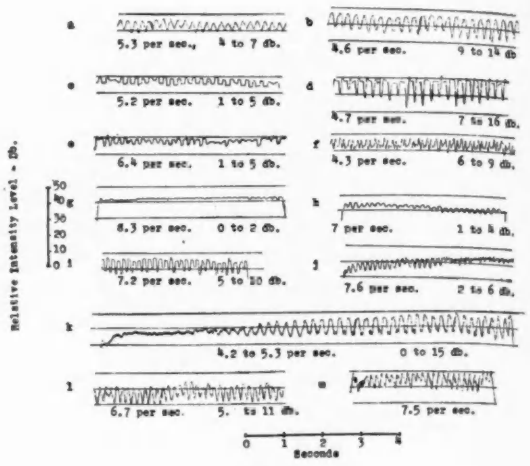
Relative Intensity Level - db.
50
40
30
20
10
0
5.3 per sec., 4 to 7 db.
4.6 per sec. 9 to 14 db
5.2 per sec. 1 to 5 db.
4.7 per sec. 7 to 16 db.
6.4 per sec. 1 to 5 db.
4.3 per sec. 6 to 9 db.
8.3 per sec. 0 to 2 db.
7 per sec. 1 to 4 db.
7.2 per sec. 5 to 10 db.
7.6 per sec. 2 to 6 db.
4.2 to 5.3 per sec. 0 to 15 db.
6.7 per sec. 5. to 11 db.
7.5 per sec.
0 1 2 3 4
Seconds

Fig. 9.

separate readings for the same individual. Fig. 9 gives some examples. Specimens *a* and *b* show medium and wide vibratos, having a relatively even form. Specimens *c* and *d* have a sort of square characteristic. Specimens *e* and *f* are quite irregular, *f* suggesting the presence of either two unphased movements or harmonics of one fundamental movement.

Specimen *g* shows a tremolo, there being no intensity variation (within the one decibel minimum indication of the recorder) until the last second or so of the tone. Specimen *i* was taken from the same voice and on this tone an almost true vibrato is found. Specimens *h* and *j* are tracings of the tremolo-vibrato, the speeds being respectively 7.0 and 7.6 per second. It appears impossible for what we term the true vibrato to occur at this frequency. Specimen *m* shows a tracing from a recording of the voice of a baritone who at one time was one of the world's greatest singers. This tracing was made with the aid of the crystal analyzer in order to permit more definite measurement of the frequency of fluctuation, which turned out to be 7.5 per second. Other measurements of the movement of this singer's voice showed that he had a true vibrato for many of his tones. It is interesting to note that a little while after this recording was made, this singer practically lost his voice.

Specimen *k* shows a tracing of a tone started softly and swelled to fortissimo on the vibrato. It will be noted that at pianissimo there is no vibrato and that, as the intensity increases, the

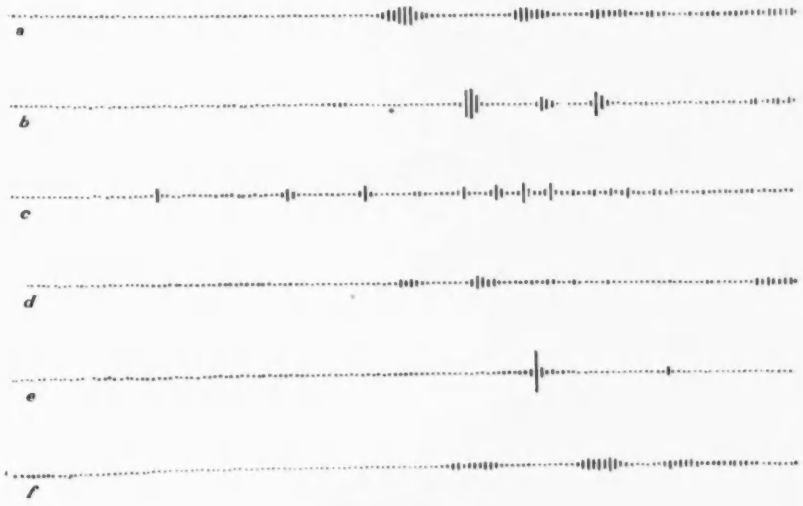

FIG. 10.

swinging of the vibrato increases until an intensity fluctuation of about 15 db is reached. Specimen *e* shows the intensity and frequency of fluctuation of a trill. The intensity change is of the order of 10 db and the frequency about 6.7 per second.

We have made measurements of the frequency of voice movements of some of the world's greatest artists, using phonograph records as our source of material. Out of 115 measurements the frequencies in the vicinity of 6.0 per second occurred thirty-one times and predominated over all other frequencies. We take this to mean that 6.0 per second is in the nature of an optimum frequency of variation, at least from the point of view of the person producing a tone. A frequency of less than about 4.5 per second does not appear as pleasing as do higher ones.

In our work we have studied the development of the vibrato in students undergoing training, and find that, as this development progresses, vibrato is introduced to a greater extent, the movement is intensified, and the frequency raised. The majority of students' readings fell between 5 and 6 per second.

The pitch variation occurring in the voice movements is also of considerable interest. In order to study this phenomenon our acoustic spectrometer was employed. The singers were requested to hold the tone and the acoustic spectrograms, shown in Fig. 10, were taken, using a three-second exposure. The deflections along the vertical axis are proportional to the sound pressure and the reed number is proportional to the pitch, along the horizontal axis.

In Fig. 10 are given specimens showing the various types of pitch fluctuations which may occur. The pitch variation which accompanies the vibrato may be very wide or relatively narrow. The gradation from wide to narrow is illustrated by specimens *a*, *b* and *c*. Specimen *d* shows the pitch variation occurring in a tremolo-vibrato. It will be observed that the pitch change is a relatively wide one. Specimen *e* shows the true tremolo frequency variation of a successful radio "crooner" of the better type of music; the frequency variation is so small as to be negligible. The pitch change occurring in the trill is a very wide one, as will be seen from specimen *f*. The trill is usually characterized by an intensity peak in the neighborhood of both high and low intensity points.

The results obtained with the spectrometer serve to indicate the relative amount of pitch variation occurring, but the absolute amount is not determined. This follows from a consideration of the physical and mathematical nature of the vibrato. In some respects, a vibrato tone is similar to what the engineer calls a frequency modulated signal, that is, it involves a tone whose frequency is varying periodically. In addition, the true vibrato also involves a periodic intensity variation. If only that part of it in the nature of a warble tone is considered, it will be recalled that

it could be represented mathematically by the sum of an infinite number of components for each partial tone, these components being separated from each other by the frequency of the vibrato, i.e., by approximately six cycles per second. The amplitude of these components, or "side-bands" is determined by Bessel's functions, involving the vibrato rate and the percentage pitch variation occurring.

Quality

With regard to singing, the term "quality" takes on several aspects. In its strict scientific sense the term is used relative to the harmonic composition of a complex tone. However, with respect to the singing voice, it has assumed more general significance and is used with reference to the total impression created by sung tones. It includes not only harmonic distribution, but also the ability to vary it, maintenance of proper pitch and intensity, and the vibrato or tremolo movement of the tone. For the present we shall be concerned with the strict physical interpretation and discuss it as a factor in singing.

The quality of a phonated tone is then determined by the number, frequency and relative intensity of its components. That a singer has some control over this is evident since he can vary the "color" and change the vowel quality. Furthermore, it may be observed that quality and vowel quality both depend on the harmonic distribution. Only a few frequencies are necessary to determine a vowel although the other partials which are present must modify the impression created, i.e., impart such characteristics as "brilliance," "shrillness," etc. It may be possible to consider that frequencies outside the vowel bands determine vowel "timbre," whereas variations in relative levels of the partials within the bands determine merely the vowel. However, this can be true only within unknown limits, and we are not at this time prepared to make such a division. Accordingly the term "vowel quality" will be employed for that general factor in singing dependent on the harmonic configuration and corresponding to the physical conception of quality. With this in mind we may proceed to a discussion of experimental results.

Harmonic analyses were made with both the crystal analyzer-level recorder combination and the acoustic spectrometer. With the former the 200 cycle band was used giving adequate separation for pitches above and including middle C (261.6 cycles per sec.). The use of the 20-cycle band is impractical because of the wide frequency spread introduced by vibrato, especially for the higher partials. A motor was used to drive the analyzer tuning control from 100 to 4600 cycles per second in ten seconds. A few readings have also been made from 4600 to 10,900 cycles, but the amount of power contained within this range is usually very small. The acoustic spectrometer observations extended from 50 to 3100 cycles per second. Photographs were taken using a three-second exposure. Because of the influence of vibrato, and the contraction of the frequency range at the upper end (the instrument has a uniform pitch scale) which causes a crowding together of the partials, the spectrometer results serve mainly as qualitative indications.

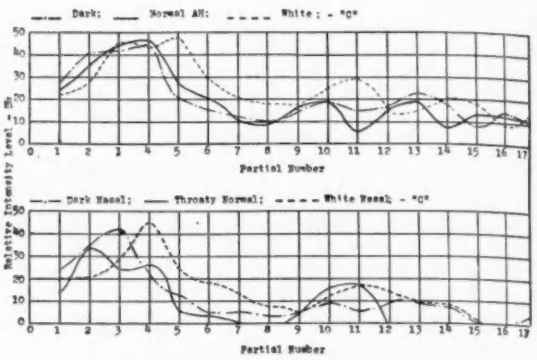
Dark; Normal AH; White; "C"
Relative Intensity Level in Db
Partial Number
Dark Nasal; Throaty Normal; White Nasal - "C"
Relative Intensity Level in Db
Partial Number

Fig. 11. Vowel "AH," "middle C," six qualities, baritone.

To lay the foundations for a thorough investigation of vowel quality observations were made on the vowel "ah" intoned in different manners. Fig. 11 is a plot of readings taken with the crystal analyzer for a baritone voice singing middle C. The partial number is plotted along the horizontal axis, and the relative intensity level in decibels along the vertical axis. In the figure continuous curves are drawn through the intensity levels of the various partials. The upper set shows the spectra for "dark," "normal" and "white" intonations. When listened to, these are heard as "aw," "ah" and "ah" with a suggestion of ă, respectively. It will be noticed that the dark quality is differentiated from the normal by a shift of energy to the lower partials in the region

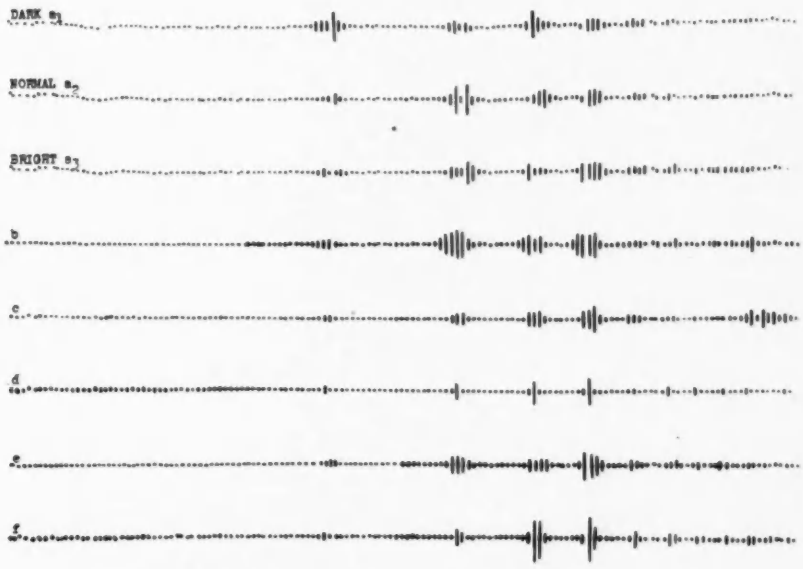

FIG. 12. Vowel "AH"—"C"—Different singers.

of the vowel determinants, whereas the reverse takes place with the white timbre. These shifts are a general occurrence and are, of course, directly traceable to the vowel characteristics. Variations in the relative intensity of the partials above the sixth (1570 cycles per second) are also apparent, but we are not yet ready to discuss their significance beyond stating that the brilliance of the tone must be affected. The lower set of curves are for dark nasal, normal and white nasal "ahs" on middle C, produced by the same singer using a purposely "shut off" technique. The so-called normal one differs from the normal one produced with an "open" technique, and a difference is audible. For the nasal tones there is present a relatively sharp peak in the determinant region. The appearance of such a peak, which does not always occur, seems to depend on the degree of nasality and the singing pitch.

When different singers are asked to sing the same vowel, marked differences in vowel quality are frequently noticeable. The question arises: Are there any general trends in the harmonic distributions of phonated tones which would indicate whether or not the vowel quality would be considered pleasing, and whether or not a singer is using his voice properly? For the former there is some affirmative evidence; for the latter our data have so far led to mainly negative conclusions. We present some typical data relative to these points.

The acoustic spectrometer was at first used to perform harmonic analyses. The types of results obtained are shown in Fig. 12. The vowel sung in all cases is "ah" on middle C. The first three, a_1, a_2 and a_3 are for a good baritone singing with dark, normal and white colors, respectively. It will be noticed that, as the vowel is brightened, there is a shift of energy upward along the pitch scale. Specimens *b*, *c* and *d* are for three baritones who sang well, fairly well, and poorly, respectively; and *e* and *f* are for two sopranos who sang well and poorly. The outstanding difference between the better and the bad singers is the lack of vibrato frequency spread which characterizes the poor voices. In regard to harmonic distribution, there is no pattern which may be selected as indicative of a good singer. For example, the good singer, *b*, has a prominent partial in the vicinity of 2500 cycles per second which is not present in a_1, a_2 and *e*. The fair singer *c* has an area of strong intensity in the vicinity of 2700 cycles, which is missing in the other voices. The poor singers, *d* and *f*, have prominent third and fourth partials, indicating a whitish quality, but the good singer, *e*, with a predominant fourth partial is whiter than either of these, and the fairly good singer, *c*, is the whitest of all the specimens.

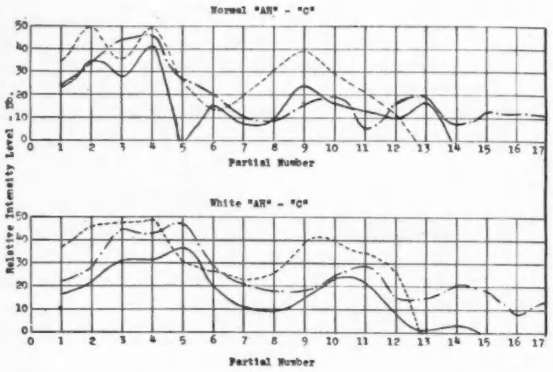

FIG. 13. Analyses of tones produced by three baritones.

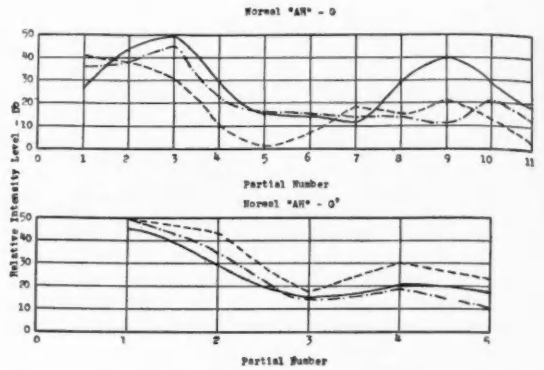

FIG. 14. Analyses of tones produced by three sopranos.

A more limited amount of data taken with the crystal analyzer is similar in import. Fig. 13 shows the analyses for normal and white "ahs" as produced by three baritones. Two of these used their voices well. Their curves are shown in dotted form. Except for the fact that the singer who produced his voice badly (solid line) gives a lower intensity than the others, there is no consistently outstanding characteristic.

In Fig. 14 are shown analyses of the voices of three women. The upper curve corresponds to the pitch G above middle C and the lower curve to the pitch G below high C. The former was produced in the lower register and the latter in the falsetto. Here again the two good singers are shown by dotted lines and the poorly-produced voice by the solid line. It will be noticed that for G there is a very pronounced area of high intensity in the vicinity of the ninth partial (3528 cycles per second) for the poor singer. This voice, in the lower register is of shrill, disagreeable character. The curves of the lower set are rather similar in contour, despite the fact that the solid curve is of a throaty poorly-produced voice, while the other two curves are of well-produced voices. At this higher pitch, G′, the vowel quality is relatively pleasing.

Our harmonic analyses represent averages over a period of time, and, as portrayed here, do not indicate the timbre variations occurring during the vibrato cycle. With the crystal analyzer the amount of time allowed for any one partial is of the order of one-half second, i.e., long enough to include about three vibrato cycles. Curves obtained using the level recorder and the crystal analyzer to indicate a few db change in level for any one partial in a vibrato cycle, but as yet we have made no attempt to determine the phase relations of these variations.

Another requirement for good singing concerns the ability of the singer to hold a constant mean pitch as long as the tone is sustained. If the pitch varies, apart from the vibrato fluctuation, the quality is not pleasant. It is not improbable that good musical quality, from the acoustical standpoint, lies primarily in accuracy of intonation and in proper vibrato variation, provided that the harmonic distribution is not abnormal.

In discussing Fig. 8 we showed how an artist could have a certain measure of success with limited vocal equipment. While this statement was not intended to depreciate the vital importance of voice, it did indicate the equally great importance of artistry. While many phases of artistry are a matter of temperament and can, therefore, hardly be measured in scientific terms, certain other phases involve mechanical skill.

Fig. 15 gives some illustrations of the use of the level recorder as an aid in studying a singer's performance.

One important factor in the interpretation of music lies in the singer's ability to start, or attack, a tone in the center of the pitch and to hold the tone constant at the desired intensity. When the tone is badly attacked, the singer starts under the pitch softly and slurs up, increasing the intensity as he raises pitch.

Specimen *e* shows a level recorder tracing of a hesitant attack. Note that the intensity starts low and rises about ten db. It then almost im-

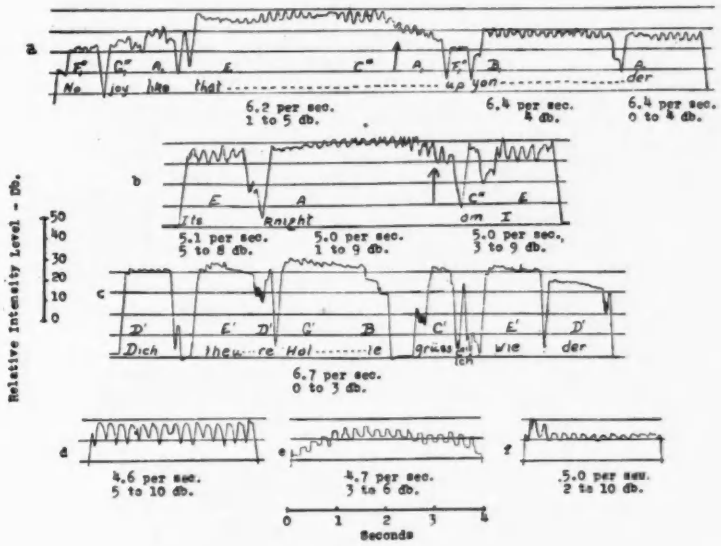

FIG. 15.

mediately starts to drop and, just before the end, the intensity is down again about 7 db. This inability to attack and hold intensity suggests a singer of inferior artistic rating and also gives the impression of a weak voice, since the full intensity is not sustained. This singer could not interpret a song with a smooth legato.

Specimen *f* shows an extremely vigorous, somewhat explosive attack. In this case the intensity of the tone, at the moment of attack, is about eight db higher than the intensity at which it is sustained. Constant use of an ultravigorous attack is inartistic, but for certain dramatic effects this device may be employed artistically. Notice that this singer is able to maintain a constant intensity. Specimen *d* shows a well attacked and well sustained tone.

Specimen *a* shows a musical phrase sung by a young man who is a good musician and who also uses his voice quite well. Notice that, except for the second tone $G_1\sharp$, and the E, which is intentionally swelled, each tone is struck at the desired intensity (or with a slightly too vigorous attack—as on the E) and maintained at a constant intensity. Notice the increase in the amplitude of the vibrato as he swells the E, and the relative smoothness and almost ideal frequency of the vibrato throughout. Notice also the vibrato run-down (see arrow) from C♯ to A_1. An inartistic or bad singer slurs from tone to tone and the vibrato disappears.

Specimen *b* shows a well-sung phrase from Wagner's *Lohengrin*. Each tone is struck squarely in the center. The high A is swelled and run down on the vibrato (see arrow). Notice that each tone rises to its proper intensity level, i.e., the A is at higher intensity than are the E's and both E's are at about the same intensity level. In specimen *a*, this relative intensity level is not always correct, since the E is no louder than the C♯. A criticism of singer *b* lies in his vibrato which is somewhat too slow.

Specimen *c* shows an inartistically sung phrase from Wagner's *Tannhäuser*. The voice is soprano. Notice the lack of proper vibrato throughout and the presence of a fluctuation at tremolo frequency. Notice that the D′ at the end is about eight db below the C′ which is nearly as loud as the E's. These deviations are not accountable for by vowel differences. There is a sag in intensity for each tone sung except the first. This indicates rendition which is poorly-balanced and lacks proper legato. Notice the absolute cessation of all movement of the voice as she slurs from G′ to B on the word "Halle," and the failure to establish the tone D′ on the syllable "re" of the word "theure." Notice the three rolled "r's."

CONCLUSION

The studies reported here represent a preliminary attempt to evaluate the mechanics of

the singing voice. The continuation of such investigations we hope will lead eventually to norms of considerable practical value and shed new light on the voice mechanism and the manner in which it functions.

We are now undertaking a series of measurements, similar to those outlined here, on some of the world's great artists. Such tests were recently made on Giovanni Martinelli. His pitch-intensity curve showed a generally uniform increase of intensity with pitch, with the levels approximating those of the reference curve. The vibrato frequency was about 5.7 per second, the shape of the intensity variation being of a smooth rounded form, amplitude of 1 to 4 db. Harmonic analysis of several tones showed the presence of strong components between 2000–3300 cycles per second, partially explaining the well-known carrying power of the voice. Individual tones were well attacked and well sustained.

Experimental Studies on the Sound and Vibration of Drum[1]

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(Received December 20, 1934)

INTRODUCTION

ALTHOUGH the vibration of a membrane has been studied very thoroughly theoretically as well as experimentally, yet in regard to the vibration of a drum, investigations have been singularly few. Besides Lord Rayleigh's[2] short note on the kettle drum and the theoretical paper by Otto Frank,[3] the only work that we know of is the experimental one of T. Terada.[4, 5] Lord Rayleigh merely remarked that the principal note heard in the kettle-drum is produced by the vibration of the membrane with one nodal diameter. Terada, on the other hand, made a series of ingenious experiments, which although of very simple character, revealed most of the fundamental properties of the vibration of a drum.

The various kinds of drums now used in Japan in connection with religious functions and in music as well as in drama, may be classified into four kinds, as follows:

(1) The simple drum with only one skin, or parchment, stretched on a frame.

(2) The ordinary cylindrical drum.

(3) The drum with a number of cord-lacings between its two ends, or faces.

(4) The drum with an hour-glass-shaped, or a dumb-bell-shaped, body.

The first three are generally called *Taiko*, corresponding to the English word "Drum," while the last, which is quite different from the rest, is called *Tudumi* or *Tsudzumi*.

All these drums are used very extensively in Japan. One of the present authors (J. O.)[6] has already studied the tone-color of the *tudumi*, a peculiar instrument having a pinched hour-glass-shaped body, with its faces therefore much larger than the body. Quite recently the authors[7] made a series of experimental investigations by means of electrical devices on the sound and vibration of a Japanese hanging-bell. The same methods were applied to the study of the vibration and sound of the drum, the results of which are contained in this paper.

METHOD OF EXPERIMENT

Of the four kinds of drums mentioned above, the one with a theoretical interest is that numbered (2), namely, the drum that consists of two parchments or membranes stretched one on each end of a cylindrical body. Two kinds of drums belonging to this class are actually in use:

(a) The *O-daiko*,[8] meaning a very large drum, with a cylindrical body two feet or more in diameter, and often used at shrines in religious functions. The length of the convexed body which has the shape of a wine cask, is usually slightly larger than the diameter. It is usually placed on a stand near the altar, and its deep rumbling tone adds solemnity to the occasion. A drum closely resembling it is used in drama.

(b) The *Turi-daiko* meaning a hanging drum, used in drama and in classical orchestral music, has a shallow body, very slightly convexed, and is hung on a frame.

The bodies of all these drums are made of hard wood, generally hollowed out of a single piece, with the membrane extending beyond the face to a short distance on the cylindrical body, and set on the circumference by one or two rows of heavy studs.

In order to study the properties of such drums,

[1] An investigation carried out with the aid of the Prince Arisugawa Memorial Fund. A résumé of this paper was published in Proc. Imp. Acad. **10**, 632 (1934).

[2] Lord Rayleigh, *Theory of Sound*, **1**, 348 (1929).

[3] Otto Frank, Sitz. Ber. d. Bayr. Akad. d. Wiss., 296 (1915).

[4] T. Terada, Proc. Tôkyô Math.-Phys. Soc. [2] **4**, 345 (1907–08).

[5] A paper entitled *Musical Drums with Harmonic Overtones* by C. V. Raman and Sivakali Kumar, which appeared in Nature **104**, 500 (1920), has little bearing on the general properties of the drum.

[6] J. Obata and Y. Ozawa, Proc. Phys.-Math. Soc. Japan, [3] **13**, 93 (1931).

[7] J. Obata and T. Tesima, Jap. J. Phys. **9**, 49 (1934).

[8] The word *Taiko* changes to *Daiko* when it is preceded by an adjective denoting its size, shape, or the purpose for which it is generally used.

The largest drum used in Japan is the *Da-daiko*, which has a diameter of more than four feet, belonging in the third or "cord-laced" class. Since it is used exclusively in classical music preserved in the Court, it plays little or no part in the life of the people.

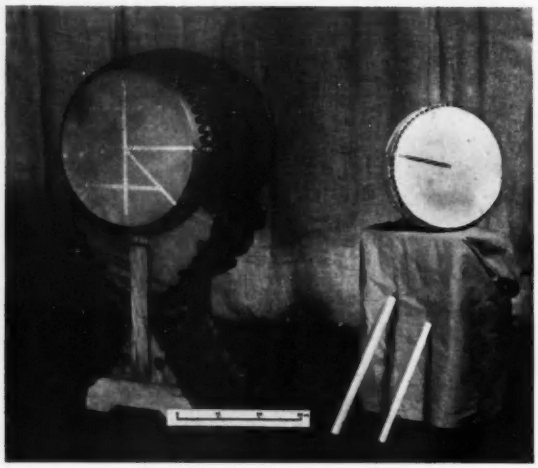

FIG. 1. (Strips of tin-foil are pasted on the membrane.)

two drums were used in our experiments. Fig. 1 is a reproduced photograph of the drums, the dimensions of which are given in Fig. 2. Besides these two actual drums, an experimental drum with a body that could be varied in length was also used, detailed descriptions of which will be given later.

Sound investigation. All the experiments were carried out in a room, having walls, floor, and ceiling covered with sound absorbents. The membrane was struck with a wooden stick, and the sound was recorded electrically with greatest possible fidelity by means of a condenser microphone made by the Western Electric Company, New York, a powerful amplifier, and an oscillograph. The microphone was placed at a distance of from 100 to 165 cm from the center of the drum.

Vibration investigation. The electrical arrangement for measuring vibrations consists of an electrical circuit containing a vacuum tube (triode), and the vibration to be measured is made to produce a change in electrical capacity C of a condenser in the circuit (Fig. 3). Since this change results in a change in the grid-voltage of the vacuum tube, there is a corresponding change in the anode current which, after suitable amplification, is recorded by means of an oscillograph.

In order that the surface of the membrane shall form one electrode of the condenser, strips of thin tin-foil were pasted on it, and as shown in Fig. 4, a parallel plate condenser was formed by a micrometer-head and the surface of the membrane; the vibration at that point was to be measured. This condenser was inserted, as described above, in the grid-circuit of a vacuum tube. It can easily be proved that, so long as the change in capacity due to the vibration is very

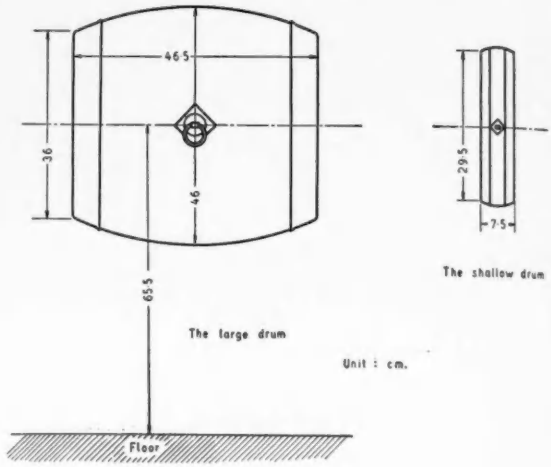

FIG. 2.

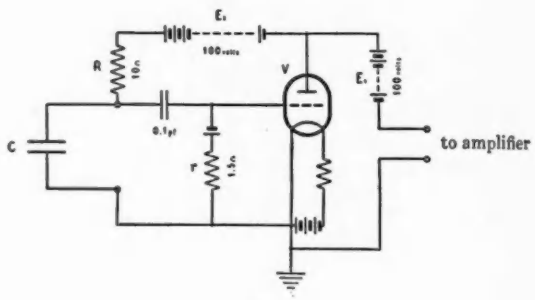

FIG. 3.

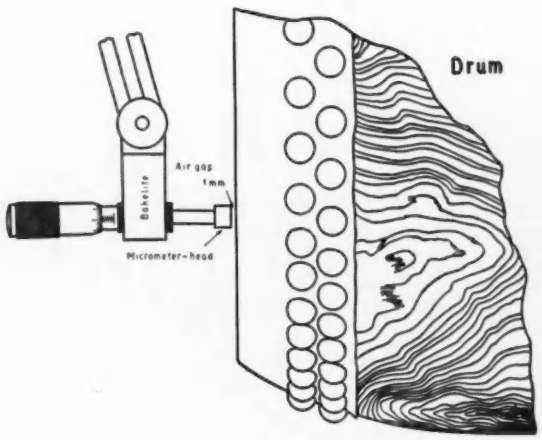

FIG. 4.

small compared with the initial capacity, the change in the grid voltage exactly corresponds to the change in capacity.

The novelty of this electrical method of measuring vibration is in its extremely high sensitivity, there being no need of bringing anything into contact with the vibrating body; and since the diameter of the micrometer-head is only ten millimeters, the record may be regarded as giving the vibration of a point.

Fig. 5 shows the complete arrangement for recording sound and vibration. For sound recording, C is a condenser microphone, while for vibration recording it is the condenser formed by the micrometer-head and the vibrating body.

Another advantage of this electrical method of measuring vibration is that as it is entirely electrical in nature, suitable electrical filters can be employed to pick up any desired partials from among the complex vibrations, an important feature in studying complex vibrations such as those of a bell or a drum.

Besides taking oscillographic records of the sound and vibration, an electrical frequency analyzer, made by the Nippon Electric Company, Tôkyô, in accordance with Mr. M. Kobayashi's design,[9] was also employed for direct analysis of the sound and vibration. This instrument which, in principle, is essentially the same as that devised by Grützmacher, is very useful in analyzing sounds or vibrations having inharmonic partials, such as those of the drum.

The microphone was carefully calibrated by means of a Rayleigh disk, and all the results of analysis were corrected for the frequency-characteristic of the microphone.

[9] M. Kobayashi, World Engineering Congress, Tôkyô (1929), Paper No. 126.

Results

The large drum

(a) *Fundamental vibration.* Two sets of vibration-recording devices, one on each membrane, were used at the same time, together with 470-cycle low pass filters. Thus, upon striking one of the membranes, the vibrations (lower partials only) of both of them were recorded on the same film.

Fig. 6a (Film No. 61) shows such a record, the timing-wave having been obtained with an electrically driven tuning-fork.

Physically speaking, such drums are nothing but a coupled-system of two vibrating membranes, coupling being effected by the air column interposed between them, and also by the wooden body in the case of the shallow drum, as will be described later. Fig. 6 is thus a beautiful example,

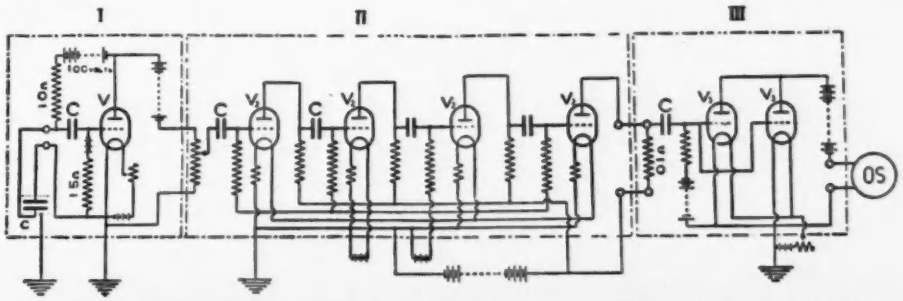

FIG. 5. Complete arrangement for recording sound and vibration. I, microphone circuit; II, voltage amplifier; III, power amplifier; C, condenser or microphone; OS, oscillograph; V_1, V_2, Marconi valve D.E. 5. b, or Tôkyô Electric Co. "240"; V_3, Tôkyô Electric Co. "171 A."

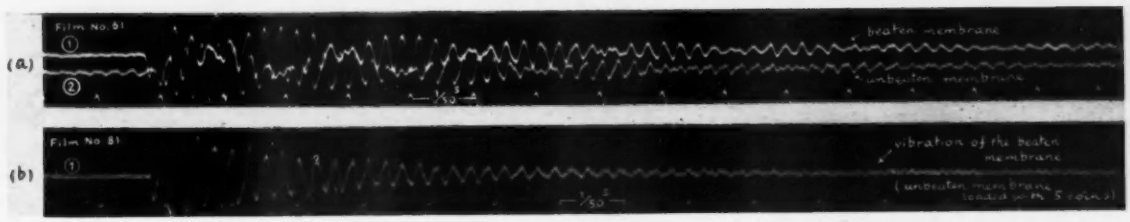

FIG. 6. Fundamental vibrations (large drum).

showing the properties of the vibration of a coupled system. The records on both faces show intermittent vibrations, the intensity of the vibration increasing alternately in each membrane. This is undoubtedly a characteristic of coupled vibration of two systems having almost equal natural periods, the energy of vibration being exchanged alternately from one system to the other.

The average pitch of the vibration was found to be 180 per second, giving 25 beats in a second. This record was taken at the end of June, the most humid time of the year in Japan. An earlier record that was taken in the middle of April, gives for the pitch of the fundamental vibration 214 per second, accompanying 25 beats in a second. This shows, that although the pitch of the vibration is affected by weather, the condition of the coupling always remains unchanged.

In coupled vibration of two membranes, two normal modes of vibration are conceivable, namely, (1) that the two membranes vibrate together, with the air between them, in the same direction, and (2) that they vibrate in opposite directions (Fig. 7). Closer inspection of vibration-

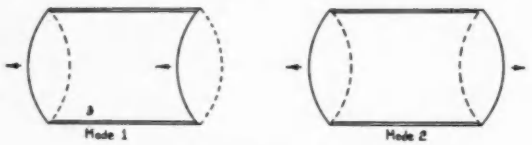

FIG. 7. Normal modes of coupled vibration.

records of the two membranes (Fig. 6a) shows that at first and at some other instant later, the two membranes vibrate in the same direction (the air-gap between the membrane and the micrometer-head being increased in one membrane and diminished in the other), while at an other instant they vibrate in opposite directions. It may thus be concluded that in the present drum, in which the length of the body is slightly larger than the diameter of the membrane, two normal modes of coupled vibration, as mentioned above, take place simultaneously, consequently causing beats. Since the average pitch of the vibration is 180 per second, giving 25 beats per second, the frequencies of modes (1) and (2) are 168 and 193 per second, respectively. Further, as will be shown later, the partial that seems to correspond to the octave of this mode was also found.

Since the sound of a drum is the result of coupled vibration of two membranes, it follows that the sound intensity on the rear or unbeaten side must be equal to that on the beaten side. This was found to be the case, as also that the sound intensity on the line that perpendicularly bisects the axis of the drum is one-half that on either of the two skin-heads of the drum [see Fig. 10].

In connection with such nature of the sound of the drum, it might be pointed out that the records of the "po" tone of the *tudumi*, reproduced in our earlier paper, clearly show a similar intermittent character. There is thus a scientific reason for the practice of *tudumi* player's frequently attaching a small piece of moistened paper to the rear (unbeaten) skin-head of the drum in order to improve its tone-color.

In order however to verify this remarkable effect of the rear membrane, records of vibration were taken after having pasted in succession one, three and then five one-sen copper coins (3.78 g per piece) to the center of the rear membrane. With only one coin, the intermittent nature of the vibration disappeared, while damping was greatly increased, the vibration dying out in almost one-third the time of the case in which the rear membrane was not loaded. [See Fig. 6b, Film No. 81.] No change, however, was observed in the second and higher partials.

(b) *Second partial. Mode having one nodal diameter.* This overtone is easily heard, being especially conspicuous when the membrane is struck near the edge.

In Fig. 8 (a) [Film Nos. 22 and 25] are reproduced the records of vibration at two points taken by striking a point 6 cm from the center of the membrane. It will be noticed that in one of the records, Film No. 22, a conspicuous beat is present, and in order to ascertain the origin of this beat, a number of records were taken at various points on the membrane, some of them being reproduced here.

Fig. 9 shows the points where the vibrations were recorded; the plain circles indicating the points where the beats were conspicuous and the black circles those where the beats were not so conspicuous. The numerals correspond to the film-number.

Thus, from the similar consideration, as that

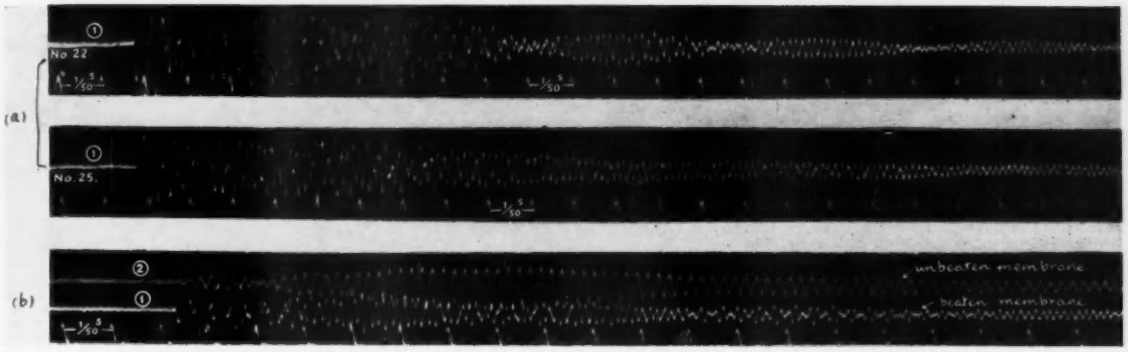

FIG. 8.

made in the case of the vibration of the Japanese hanging bell described in an earlier paper,[10] it seems justifiable here to attribute the origin of this beat to the fact that, because of lack of uniformity in the thickness of the membrane and in the tension at different parts of the membrane, the second partial that appeared to have only one nodal diameter consists in reality of two like vibrations, the nodal line consequently becoming a pair of hyperbolas. Undoubtedly, this characteristic is peculiar to the present drum.

As the average pitch of the vibration was found to be 333 per second, accompanying 11 beats per second, the pitches of the component vibrations were 339 and 328 per second, respectively. Damping is very slight compared with the fundamental vibration; the vibration lasting for a fairly long period. This may be accounted for by the fact, that as this mode has only one nodal diameter, the air inside undergoes no change in density, with the result that the other membrane is scarcely affected, both these causes resulting in very little dissipation of energy.

Further, in order to make clearer the effect of the other (unbeaten) membrane on the second and higher partials several records were taken by using two sets of vibration-recording devices one on each membrane. One of the micrometer-heads was placed on the point on the beaten side, where the second partial was remarkably excited, i.e., the point 10 cm from the center [the same point as that for Fig. 7 (a), Film No. 17]. The other micrometer-head was placed on various points on the unbeaten side along the circle 10 cm from the center. Thus, the vibrations of both membranes were recorded on the same film; one of the records thus obtained being reproduced in Fig. 8 (b). It will easily be observed from this record that the vibration excited in the unbeaten side consists mainly of the fundamental mode, that is to say, the coupling between the two membranes is made only for the fundamental mode of vibration.

(c) *Higher partials.* A point 6 cm away from the center of the membrane was struck quickly in succession, and the sound, picked up by a condenser microphone set at a distance of 165 cm from the center of the drum, was analyzed by means of the electrical frequency analyzer. The record taken by the analyzer is reproduced in Fig. 10. The data given in Table I are the means of several determinations obtained from a number of similar records.

As regards the relative intensity, the funda-

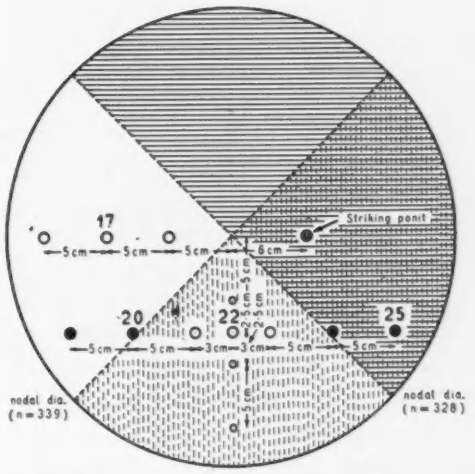

FIG. 9. Modes of vibration of the second partial.

[10] J. Obata and T. Tesima, reference 7, p. 64.

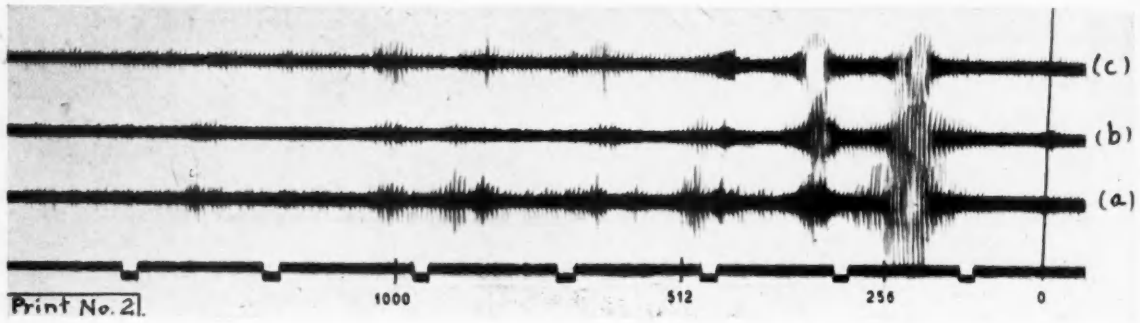

FIG. 10. Directivity of the sound intensity. Membrane (2) was struck at a point 6 cm from the center. Microphone was placed at a distance 165 cm from the drum-center: (a) facing the unbeaten membrane; (b) facing the beaten membrane; (c) on the line which bisects the axis of the drum perpendicularly.

mental is the strongest, the second partial ranks next, while the other higher partials are comparatively small.

At least in the lower partials, all those correspond to the partials theoretically known in connection with the free vibration of a stretched membrane were observed in the case of the drum, the pitch differing but little from that of free vibration.

The third partial having a pitch 383 per second, however, should be distinguished from the others. This partial seems to be the octave of the fundamental vibration: the two membranes vibrate together, forming a transverse node in the middle of the air-column. If we assume the drum to be an open pipe, the length of whose body is 46.5 cm, the pitch of this mode will evidently be

$$N = 340/(46.5 \times 2) = 365 \text{ per second.}$$

This agrees fairly well with 383, the actual value obtained for the drum, which admittedly differs somewhat from an open pipe.

Further, by using two sets of vibration-recording devices at the same time, and striking the drum at a point 6 cm from the center of the membrane the vibrations at the center and at the rim were compared by recording on the same film. It was found that at the center the fundamental vibration is, as it should be, very conspicuous, the second partial being entirely absent, while at a point near the rim the second partial is very conspicuous. The mode corresponding to the octave of the fundamental was observed only at the center.

TABLE I. *Pitch of the partials.* (Large drum.)

Obs. pitch	—Relative Value— Obs.	Theor. value for single membrane		Remark
214 (Fund.)	1.00	1.00	◯	Intermittent vibration
333	1.61	1.59	⊖	Accompanies beat
383	1.79	2.00		
468	2.19	2.13	⊕	Octave of the fundamental
492	2.30	2.29	◎	
544	2.54	2.65	⊗	
621	2.91	2.92	◎	
695	3.25	3.15	⊗	
739	..	..		
865	..	..		
905	..	..		
1023	..	..		
1188	..	..		
1296	..	..		
..	..	..		

The shallow drum

The results given in Table II were obtained for the sound and vibration of the shallow (short) drum by experiments made exactly in the same way as in the case of the large drum.

The foregoing result shows that so far as the pitch of the partials is concerned, no difference was observed between these two kinds of drum. A conspicuous difference, however, is seen in the lack of beats in the fundamental vibration in the shallow drum (Fig. 11). Oscillograms showed that the second partials of the two membranes have

TABLE II. *Pitch of the partials.* (Shallow drum.)

Obs. pitch	—RELATIVE VALUE— Obs.	Theor. value for single membrane		Remark
195 (Fund.)	1.00	1.00	○	
310 {295 {324	1.59	1.59	⊖	Accompanies beat
440	2.26	{2.13 {2.29	⊕ ◎	Two modes mixed
478	2.45	2.65	✳	
592	3.03	2.92	⊕	
743	3.80	..		
1080	5.54	..		

almost the same frequency. It was therefore naturally expected that the fundamentals of the two membranes would also have like frequencies with consequent intermittent vibrations, which, however, did not turn out to be the case.

The reason for the absence of intermittent vibrations from the shallow drum may be explained by the closeness of coupling compared with that in the large drum. The length of the body, which is only 7 cm in this drum is very small compared with the diameter, which is 29 cm. Moreover, as will be shown later, the coupling is still further increased by the vibration of the body itself, which is quite conspicuous in this case.

In a shallow drum, the manner in which it is supported, whether it rests on a stand or is hung on a frame, gives rise to considerable difference in tone-color, which is reflected in the effect of the vibration of the body itself on the tone-color. Closer studies of this effect [see Fig. 12, Print 14] showed that by hanging the drum, instead of resting it on a stand, only the fundamental vibration undergoes considerable change, while the higher partials suffer no perceptible effect. That is to say, by hanging the drum, the pitch of the fundamental was increased from 195 to 206, and damping was reduced, which shows that when the drum rests on a stand or on the floor, the vibration of the body is prevented and consequently much more energy is dissipated therein. In order to verify this, radial as well as axial (i.e., in the direction parallel to the axis) vibrations of the body were studied by pasting a strip of tin-foil to the edge of the body and by utilizing the electrical frequency analyzer. The results clearly show the presence of a remarkable amount of vibration in the body corresponding to the fundamental of the membrane.

Effect of the length of the body

Finally, in order to study the effect of the body-length, an experimental drum, as shown in Fig. 13, was made. The central cylindrical part was made in three different lengths, namely, 9,

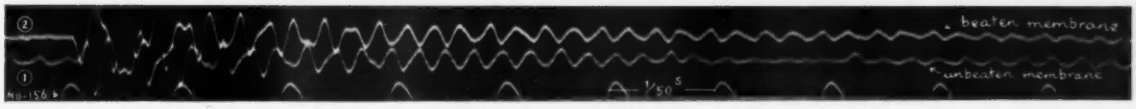

FIG. 11. Fundamental vibrations (shallow drum).

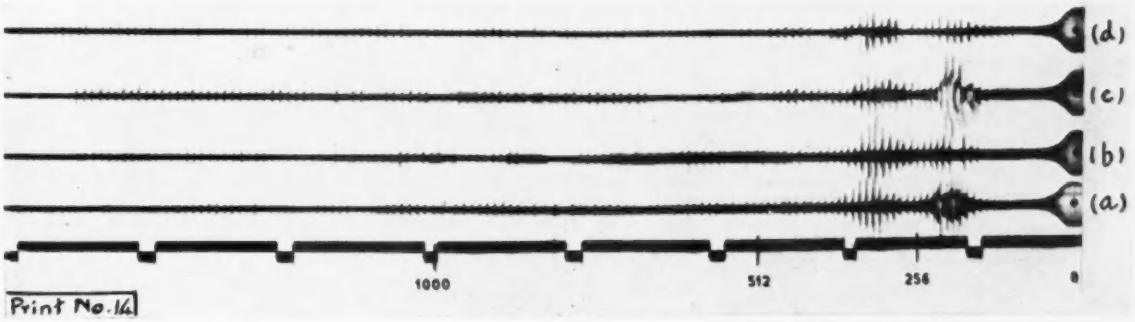

FIG. 12. Effect of hanging. (a), (c), hung; (b), (d), supported. (a), (b), the membrane (2) was struck, (c), (d), the membrane (1) was struck; the microphone always facing the beaten membrane at a distance 1 m from the drum-center.

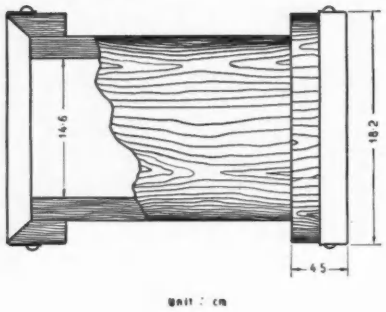

FIG. 13.

14.5 and 24.5 cm. One end of the cylinder was always closed by the membrane ②, while the condition of the other end was changed in three different ways, namely, (a) it was closed by the membrane ①; (b) was open; (c) was closed with a hard wooden plate.

The effect of body-length was studied for these three cases by striking the membrane ② at a point one-third the radius from the center. Because of the simple construction of the drum the diameter of the air-column is slightly larger at the end than at the central part, which, however, seems to be practically immaterial.

The results from a large number of records of sounds as well as of vibrations obtained by means of the oscillograph and the electrical frequency analyzer, show that in every case, regardless of the condition of the other end, only the pitch of the fundamental is lowered by increasing the length of the body, the higher partials being unaffected. Indeed, the effect is more conspicuous, which is as it should be, when the other end is either open or when it is closed with a hard wooden plate than when it is closed with membrane.

SUMMARY AND CONCLUSIONS

The vibration as well as the sound of the drum was investigated by means of electrical devices that gave faithful records of the vibration and sound. To analyze them directly an electrical frequency analyzer was also used. The principal results obtained were as follows:

(1) The vibration, consequently the sound, of a drum consists of two parts, namely, (a) coupled vibration of the fundamental vibrations of the two membranes, (b) the normal modes of vibration of the struck membrane.

(2) In the coupled vibration, each membrane vibrates as a whole, coupling being effected mainly by the air-column, and, in the case of a shallow drum, also by the wooden body.

(3) In a drum of comparatively long body and possessing a rich tone, the fundamentals of the natural vibrations of the two membranes have equal or nearly equal pitch, in consequence of which their coupled vibration takes the form of intermittent vibrations in both membranes. The fact that in a shallow drum no such intermittent vibration is observed may be explained by the closeness of coupling in this case. In either cases the sound intensity therefore differs little in front or in the rear of the drum.

(4) An octave of the fundamental vibration, in which the two membranes vibrate together, forming a transverse node in the middle of the air-column, was also observed in the case of the large drum.

(5) Among the normal modes of vibration, that having only one diametral nodal line is the most conspicuous. In the drum used in our present investigations, at any rate, this mode, perhaps because of non-uniformity in thickness and tension of the membrane, consists of two like vibrations, resulting therefore in beats.

(6) The pitch of the fundamental tone is lowered by increasing the length of the body, the higher partials being unaffected.

In conclusion, the authors wish to express their sincere thanks to Mr. Yahei Yosida for his able assistance in some parts of the experiments.

Sound Waves of Finite Amplitude in an Exponential Horn

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1. INTRODUCTION

THE recent use of horn loudspeakers to give large low frequency output[1] makes it desirable to investigate the production of alien tones in the horn due to nonlinear action of the medium. This problem was treated first by M. Y. Rocard,[2] but owing to confusion between Lagrangian and Eulerian methods, his general equation and that for the exponential horn are incorrect. The present paper shows that M. Rocard's analysis gives a value for the power associated with the second harmonic which is *four* times too large, i.e., the calculated value is 6 db too high. This figure has been found experimentally by Thuras, Jenkins and O'Neill,[3] so our analysis has confirmation.

2. SOLUTION OF HORN EQUATION

For brevity we omit the derivation of the equation for finite amplitude. In the following we take Eq. (11) from Chapter XI in *Loudspeakers*[4] and use the symbolism therein. The equation to be solved is

$$\xi''+\beta\xi'(1+\xi')=(\ddot{\xi}/c^2)(1+\xi')^{\gamma+1}e^{(\gamma-1)\beta\xi}, \quad (1)$$

where ξ is the particle amplitude at abscissa x measured from the throat as origin, $\xi'=\partial\xi/\partial x$, $\xi''=\partial^2\xi/\partial x^2$ and $\ddot{\xi}=\partial^2\xi/\partial t^2$.

By expanding the right-hand side of (1) and discarding terms of higher orders than ξ^2, we obtain the approximate equation

$$\xi''+\beta\xi'-\frac{\ddot{\xi}}{c^2}=\frac{\ddot{\xi}}{c^2}[(\gamma-1)\beta\xi+(\gamma+1)\xi']-\beta\xi'^2, \quad (2)$$

where $\xi'\ll 1$ and $(\gamma-1)\beta\xi\ll 1$.

To solve (2) we use a method of successive approximation, so in the first instance the right-hand side can be neglected. Thus we have

$$\xi''+\beta\xi'-\ddot{\xi}/c^2=0. \quad (3)$$

At the throat $x=0$ and $\xi=\xi_0\cos\omega t$, ξ_0 being the maximum particle amplitude, so the solution of (3) for the transmitted wave alone, is

$$\xi=\xi_0 e^{-\frac{1}{2}\beta x}\cos(\omega t-\alpha x), \quad (4)$$

which is the expression for infinitesimal amplitudes with $\alpha^2=k^2-\frac{1}{4}\beta^2$. To find a second approximation we substitute from (4) in the right-hand side of (2), and after some reduction, we get

$$\xi''+\beta\xi'-\ddot{\xi}/c^2 = -\tfrac{1}{2}\xi_0^2 e^{-\beta x}[A_1+B_1\cos 2\theta+C_1\sin 2\theta], \quad (5)$$

where $\theta=(\omega t-\alpha x)$; $A_1=\frac{1}{2}\beta k^2(\gamma-1)$;

$$B_1=\tfrac{1}{2}\beta\{k^2(\gamma-5)+\beta^2\};\ C_1=\alpha\{k^2(\gamma+1)-\beta^2\}.$$

To find a particular integral of (5), put $\xi=we^{-\beta x}$ and we obtain

$$w''-\beta w'-\ddot{w}/c^2 = -\tfrac{1}{2}\xi_0^2[A_1+R\{(B_1-iC_1)e^{2i\theta}\}], \quad (6)$$

R indicating that the real part is to be taken.

A particular integral of (6) is

$$w=(A_1/2\beta)\xi_0^2 x+\xi_0^2 R(F_1 e^{2i\theta}), \quad (7)$$

where $F_1=D_1-iE_1$;

$$D_1=(1/8\beta)\{2k^2(\gamma+1)-\beta^2\gamma\};\ E_1=\tfrac{1}{4}\alpha(2-\gamma).$$

To (7) must be added a complementary function of (5) so that when $x=0$, $\xi=0$, since the motion at the throat is specified by (4) when x is zero. The complementary function in question is a multiple of the real part of $e^{\frac{1}{2}\beta x}e^{2i\theta_1}$, where $2\theta_1=(2\omega t-lx)$ and $l^2=(4k^2-\frac{1}{4}\beta^2)$. Thus we obtain,

$$w=(A_1/2\beta)\xi_0^2 x+\xi_0^2 R\{F_1(e^{2i\theta}-e^{(\frac{1}{2}\beta x+2i\theta_1)})\}. \quad (8)$$

Since $\xi=we^{-\beta x}$, we have from (8) on substituting from above for A_1,

[1] See a paper by Wente and Thuras, Bell Sys. Tech. J. **13**, 259 (1934), where the speakers in the Philadelphia-Washington Auditory Perspective demonstration were rated to deliver 30 watts at 200~.

[2] Comptes rendus **196**, 161 (1933).

[3] Reference 1, p. 265, and footnote 9, p. 277.

[4] McLachlan (Clarendon Press, Oxford 1934), also Wireless Eng. **11**, 27 (1934).

$$\xi=\tfrac{1}{4}\xi_0^2k^2(\gamma-1)xe^{-\beta x}+\xi_0^2e^{-\frac{1}{2}\beta x}\times R\{F_1(e^{2i\theta-\frac{1}{2}\beta x}-e^{2i\theta_1})\}. \quad (9)$$

The complete solution of (1) to the second approximation, is the sum of (4) and (9), so

$$\xi=\xi_0e^{-\frac{1}{2}\beta x}\cos\theta+\xi_0^2e^{-\beta x}[D_1\cos 2\theta+E_1\sin 2\theta]-\xi_0^2e^{-\frac{1}{2}\beta x}[D_1\cos 2\theta_1+E_1\sin 2\theta_1]+\tfrac{1}{4}\xi_0^2k^2(\gamma-1)xe^{-\beta x}. \quad (10)$$

In the solution of (5) there is also a complementary function $(1-e^{-\beta x})$ which is independent of t and vanishes at $x=0$. Any multiple of this can be added to (10). If $(C_0/\beta)(1-e^{-\beta x})$ is added to (10) and $C_0\rightarrow C_1$ as $\beta\rightarrow 0$, the extra term is C_1x. The latter is the solution of Eq. (1) in Chapter XI of *Loudspeakers*, namely, $\rho/\rho_0=1/z'\chi$ when ρ is constant, as it would be for unidirectional displacement in a long tube ($\beta\rightarrow 0$). Thus in Lamb's solution for a tube,[5] *any* multiple of x may be added (since ξ'' and $\ddot{\xi}$ are then both zero), and this accounts for the difference between his constant and that in (10) when $\beta\rightarrow 0$. In this respect the mathematical problem for the horn or the tube, without initial conditions, is indeterminate.

3. Sound Pressure

By using (17) from *Loudspeakers* (Chapter XI), the excess pressure at variable abscissa $x+\xi$ is

$$p=\rho_0c^2\{-\beta\xi-\xi'+\beta\gamma\xi\xi'+\tfrac{1}{2}\beta^2\gamma\xi^2+\tfrac{1}{2}(\gamma+1)\xi'^2\}, \quad (11)$$

provided $\beta\gamma\xi\ll 1$, $\xi'\ll 1$ and terms of order higher than ξ^2 are discarded. Substituting in (11) for ξ, ξ' and ξ'' from (10) and rejecting terms of higher orders than ξ^2, we find that the excess pressure at $x+\xi$ is

$$p=-\rho_0c^2ke^{-\frac{1}{2}\beta x}\{\xi_0\sin(\theta+\epsilon_1)+(\xi_0^2/k)\sqrt{(M_1^2+M_2^2)}\sin(2\omega t+\epsilon_2)\}+\tfrac{1}{2}\rho_0c^2\xi_0^2k^2e^{-\beta x} \quad (12)$$

where

$$M_1=e^{-\frac{1}{2}\beta x}[G_1\cos 2\alpha x+H_1\sin 2\alpha x]-[J_1\cos lx-K_1\sin lx];$$

$$M_2=e^{-\frac{1}{2}\beta x}[-G_1\sin 2\alpha x+H_1\cos 2\alpha x]+[J_1\sin lx+K_1\cos lx];$$

$$M_1^2+M_2^2=(J_1^2+K_1^2)+e^{-\beta x}(G_1^2+H_1^2)-2e^{-\frac{1}{2}\beta x}[(J_1^2+K_1^2)(G_1^2+H_1^2)]^{\frac{1}{2}}\times\sin\{(2\alpha-l)x+\epsilon_3\};$$

$$\doteq(J_1^2+K_1^2)=4k^2(D_1^2+E_1^2),$$

when x is large enough and $e^{-\frac{1}{2}\beta x}\ll 1$.

$$\epsilon_1=\tan^{-1}\beta/2\alpha;\ \epsilon_2=\tan^{-1}M_2/M_1\doteq -2kx,$$

by using the data in §7 when $x>200$ cm;

$$\epsilon_3=\tan^{-1}(G_1J_1-K_1H_1)/(G_1K_1+J_1H_1);$$

$$G_1=2\alpha D_1-\tfrac{1}{4}\alpha\beta(\gamma-1);$$

$$H_1=\tfrac{1}{4}(\gamma+1)(k^2-\tfrac{1}{2}\beta^2)-2\alpha E_1;$$

$$J_1=lD_1+\tfrac{1}{2}\beta E_1;\quad K_1=lE_1-\tfrac{1}{2}\beta D_1.$$

4. Plane Waves in a Long Uniform Tube

When $\beta\rightarrow 0$ we have the case of plane waves in a frictionless tube without reflection, and equation (1) reduces to the well-known form

$$\xi''=(\ddot{\xi}/c^2)(1+\xi')^{\gamma+1}, \quad (13)$$

whose solution[5,6] can be obtained from (12) when $\beta\rightarrow 0$. Thus the pressure at abscissa $x+\xi$ is

$$p=-\rho_0c^2k\{\xi_0\sin(\omega t-kx)-\tfrac{1}{4}\xi_0^2k^2(\gamma+1)[x^2+1/4k^2]^{\frac{1}{2}}\sin(2\omega t+\epsilon_2)\}+\tfrac{1}{2}\rho_0c^2\xi_0^2k^2, \quad (14)$$

where $\epsilon_2=-2kx-\tan^{-1}(1/2kx)$.

From (14) it is seen that the amplitude ratio of the second harmonic to the fundamental steadily increases with increase in x, the distance from the diaphragm, since there is no expansion to limit its growth as in the case of the flaring horn. Ultimately we presume that a discontinuity occurs at a certain distance x_1. As in the case of the horn, the total power in the tube apparently increases with increase in x, but this is the method of solution explained in §6.

[5] *Dynamical Theory of Sound*, p. 180, Eq. (30), edition (1910).

[6] This equation has been solved in an entirely different way by R. D. Fay. See J. Acous. Soc. Am. **3**, 222 (1931).

5. Particle Velocity

In the horn the particle velocity at variable abscissa $x+\xi$ is from (10),

$$\dot{\xi}=-\omega e^{-\frac{1}{2}\beta x}\{\xi_0 \sin\theta + 2\xi_0^2\sqrt{(N_1^2+N_2^2)}\sin(2\omega t+\psi_2)\}, \quad (15)$$

where

$$N_1=e^{-\frac{1}{2}\beta x}[D_1\cos 2\alpha x - E_1\sin 2\alpha x] - [D_1\cos lx - E_1\sin lx];$$

$$N_2=e^{-\frac{1}{2}\beta x}[-D_1\sin 2\alpha x - E_1\cos 2\alpha x] + [D_1\sin lx + E_1\cos lx];$$

$$N_1^2+N_2^2=(D_1^2+E_1^2)\{1+e^{-\beta x} - 2e^{-\frac{1}{2}\beta x}\cos(2\alpha-l)x\};$$

$$\doteq (D_1^2+E_1^2),$$

when x is large enough and $e^{-\frac{1}{2}\beta x}\ll 1$.

$$\psi_2=\tan^{-1}N_2/N_1\doteq -2kx,$$

by using the data in §7 when $x>200$ cm.

In a uniform tube $\beta\to 0$ and (15) degenerates to

$$\dot{\xi}=-\omega\{\xi_0\sin(\omega t-kx) - \tfrac{1}{4}\xi_0^2k^2(\gamma+1)x\sin(2\omega t-2kx)\}. \quad (16)$$

6. Relative Power in Second Harmonic

In evaluating the power transmitted down the horn, we can imagine the air particles over the section to be represented by a massless vibrating lamina whose central position is abscissa x, but whose area is $A=A_0e^{\beta(x+\xi)}$, where ξ is given by (10). The alternating sound pressure on the fictitious diaphragm is given by (12), and the velocity by (15). The power associated with the diaphragm is the mean value of $p\dot{\xi}A_0e^{\beta(x+\xi)} = p\dot{\xi}A_0e^{\beta x}e^{\beta\xi}$ over a complete cycle of the fundamental. When x is large enough $\beta\xi\ll 1$, so $e^{\beta\xi}$ can be taken as unity, and we have to determine the mean value of $p\dot{\xi}A_0e^{\beta x}$. Since the amplitudes in (12) and (15) are maximum values, we obtain

$$P=\tfrac{1}{2}\rho_0cA_0\omega^2\xi_0^2\cos\epsilon_1 \times\{1+(2\xi_0^2/k)[(M_1^2+M_2^2)(N_1^2+N_2^2)]^{\frac{1}{2}} \times\cos(\epsilon_2-\psi_2)/\cos\epsilon_1\}, \quad (17)$$

$$=P_1(1+\varphi_h), \quad (18)$$

where P_1 is the power in the fundamental, and

$$\varphi_h=(\text{power in second harmonic})/P_1 = (2\xi_0^2/k)[(M_1^2+M_2^2)(N_1^2+N_2^2)]^{\frac{1}{2}} \times\cos(\epsilon_2-\psi_2)/\cos\epsilon_1. \quad (19)$$

The first term in (17) represents the power supplied to the horn by the diaphragm at the fundamental frequency, whilst the second gives the power in the harmonic.[7] It appears, therefore, that the total power in the horn increases with increase in distance from the throat. This apparent paradox is due to the method of approximation used in solving Eq. (1), and can be explained as follows: If the approximation is carried further and terms in ξ_0^3, ξ_0^4, etc., are included in (10), terms involving θ, 2θ, 3θ, etc., are introduced. The net result is that as x increases from the throat, the pressure and particle velocity of the fundamental steadily get less than the values given by (12) and (15), the amplitudes approaching limiting values asymptotically. As the sound waves travel down the horn, the power lost by the fundamental is transferred to the harmonics, but to show this effect analytically would entail a considerable amount of tedious algebra. It is felt that the present analysis is adequate for immediate needs. After the waves reach a certain distance from the throat, the transfer of power to the harmonics is substantially complete. That is to say, further transfer is prevented because of increase in the rate of expansion of area. The ratio φ_h is then greatest and can readily be evaluated by taking x large enough to make $e^{-\frac{1}{2}\beta x}\ll 1$. Under this condition terms containing $e^{-\frac{1}{2}\beta x}$ in the expressions for M_1, M_2, N_1, N_2 are negligible, and we obtain

$$\varphi_h=\tfrac{1}{4}\xi_0^2\{(k^4/\beta^2)(\gamma+1)^2-k^2(5\gamma-4) + \beta^2(\gamma-1)\}\cos(\epsilon_2-\psi_2)/\cos\epsilon_1. \quad (20)$$

Since $\epsilon_1=\tan^{-1}\beta/2\alpha$ and $(\epsilon_2-\psi_2)=\tan^{-1}\beta/2l$ when $e^{-\frac{1}{2}\beta x}\ll 1$, it follows that $\cos\epsilon_1=\alpha/k$ and $\cos(\epsilon_2-\psi_2)=l/2k$. Thus (20) can be written[8]

---

[7] Since the total power is actually P_1, the ratio $P_2/P_1=\varphi_h/(1-\varphi_h)$, but since $\varphi_h\ll 1$, this refinement is unnecessary.

[8] *Note added March 11, 1935:* Formula (20) was given by the authors in Wireless Eng. **11**, 427 (1934). In J. Acous. Soc. Am. **6**, 173 (1935), Messrs. Thuras, Jenkins and O'Neill give a formula for p_2/p_1 [(19) in their paper]. Their analysis differs from that herein in several respects:

$$\varphi_h=(\xi_0^2 l/8\alpha)\{(k^4/\beta^2)(\gamma+1)^2 - k^2(5\gamma-4)+\beta^2(\gamma-1)\}, \quad (21)$$

provided $e^{-\frac{1}{2}\beta x}\ll 1$.

Calculations respecting the power in the second harmonic must of course be limited to keep $\varphi_h\ll 1$.

It may be remarked that either p_2^2 or $\dot{\xi}_2^2$ might have been taken as an index of the power in the harmonic. This procedure is not general, and is only justifiable if p_2 and $\dot{\xi}_2$ have the same phase relationship as p_1 and $\dot{\xi}_1$, i.e., $(\epsilon_2-\psi_2)=\epsilon_1$.

7. Numerical Example

We take data from the paper by Wente and Thuras,[1] excepting that to illustrate the analysis more vividly, a throat radius of 0.85 cm is chosen instead of 10 cm used in the tests. The cut-off is $32\sim$ giving $\beta\doteq 0.012$, whilst the power radiated at $200\sim(k=0.037)$ is 30 watts. Since the power in the fundamental is $P_1=\frac{1}{2}\rho_0 c A_0\omega^2\xi_0^2\cos\epsilon_1$, then $\xi_0^2=2P_1/\rho_0 c^3 k\alpha A_0$, where A_0 is the throat area. Thus from (21)[9]

$$\varphi_h=P_1 l\{(k^4/\beta^2)(\gamma+1)^2-k^2(5\gamma-4) +\beta^2(\gamma-1)\}/4\rho_0 c^3 k\alpha^2 A_0, \quad (22)$$

$$\doteq 1.48\times 10^{-4}(P_1/A_0)(n/n_c)^2; \quad (23)$$

where $n=$ transmitted frequency, $n_c=$ cut-off frequency, $k\gg\frac{1}{2}\beta$ and $e^{-\frac{1}{2}\beta x}\ll 1$. This formula can be applied to the above data, provided $x>200$ cm, so $10\log_{10}\varphi_h$ has been plotted in Fig. 1a. The ultimate level of the second harmonic is 11.2 db below that of the fundamental. For high quality reproduction a level of -30 db is desirable. In this case $\varphi\leq 10^{-3}$, so the throat area

$$A_0\geq 0.148 P_1(n/n_c)^2, \quad (24)$$

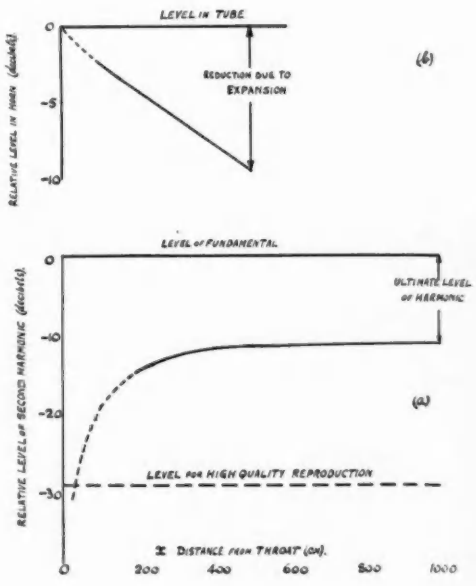

FIG. 1.

which gives 173 cm², this corresponding to a diameter of about $5\frac{7}{8}$ inches.

8. Influence of Flaring in Reducing Second Harmonic

To illustrate the reduction in the second harmonic due to expansion, we compare its power in the horn with that in a long uniform tube. For the latter, the ratio of the power in the harmonic to that in the fundamental, from (14) and (16), together with the relationship $\xi_0^2=2P_1/\rho_0 c^3 k^2 A_0$, is

$$\varphi_t=5.14\times 10^{-12}P_1 x^2 n^2/A_0, \quad (25)$$

provided $x\gg\frac{1}{2}k$, and $x>200$ cm. To compare the distortion in the horn with that in the tube, we take $10\log_{10}(\varphi_h/\varphi_t)$, which gives the level of the horn harmonic below that in the tube at various distances from the diaphragm. The result is shown in Fig. 1b, where the effect of expansion is to reduce the second harmonic about 9.5 db at a distance of 500 cm from the throat. The comparison cannot be accurately taken beyond this point, owing to the degree of approximation of the analysis. It is clear, however, that for a given power transmitted, the higher the cut-off the greater the rate of expansion at any specified abscissa, and the smaller the relative value of the second harmonic thereat.

(1) The second harmonic pressure at $x=0$ is taken as zero, whereas Lamb's formula, upon which the analysis is based, requires that $\xi_2=0$ at $x=0$. It can be deduced from (12) herein, that p_2 is not zero at $x=0$; (2) in deriving their formula, x and k are not restricted, whereas actually x must be large and $k\gg\frac{1}{2}\beta$. With these restrictions, this formula reduces to that obtained from the present analysis.

[9] In (23), (24) and (25), the power P_1 is expressed in watts.

An Apparatus for Projecting Harmonic Curves in Space

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(Received January 20, 1935)

THREE simple harmonic motions, when compounded along the x, y and z axes, respectively, yield curves of extreme grace and beauty. The behavior of these curves, as phase changes occur, is quite bewildering in its complexity, three different phase changes being simultaneously possible.

The author wishes to stress the simplicity of the mechanism necessary for showing a limited group of these curves. The essentials consist of a white enamelled pencil mounted in the flat end of a motor pulley so as to sweep out a cylinder as the motor runs, and a paper card having a single slit at the center which is driven back and forth by any vibrating mechanism, in the slide block of a stereopticon. The vibrating card causes the lantern to project a sheet of light in space, the sheet of light flickering one way and another with simple harmonic motion. When the cylinder generated by the revolving pencil intersects the sheet of light, curves such as those pictured in Figs. 1 and 4 are seen. The curves are all sine curves wrapped around a cylinder. The circular motion represents in itself the combination of two simple harmonic motions of the same frequency and fixed phase difference of ninety degrees, while the vibrating slit adds the third component motion. Phase creep between this third motion and the other two causes a rotation of the figure. It flows around the cylinder without changing shape. Altering the frequency of vibration of the card with respect to the frequency of revolution of the pencil will change the shape of the curve.

The curves in general correspond to the parametric equations,

$$x = A \sin 2\pi N_x t,$$
$$y = B \sin (2\pi N_y t + \phi_y),$$
$$z = C \sin (2\pi N_z t + \phi_z),$$

where A, B and C are amplitude constants, N is frequency, t is time, and ϕ is the phase angle. The cylindrical family of curves result when A and B are equal, N_x equals N_z, and ϕ_z is $\pi/2$. In Fig. 1 the frequency ratios are 1–2–1. In Fig. 4 they are 1–4–1. Synchronous motors were used for driving pencil and card. The time of exposure was three minutes. The sharpness of the pictures gives an excellent proof of the time-keeping characteristics of these motors. A motor that "hunts" will cause the curve to oscillate. Frictional load on one motor will cause a slight rotation of the curve to a different fixed position. Inductance or capacitance introduced into either motor circuit will give similar effects, the directions of rotation being opposite. Change in power factor may be found directly by measuring the angle of rotation with a radially mounted telescope. Excessive frictional load will cause a small motor to slip poles, the curve rotating in uneasy jumps. The ear soon learns to judge from the sound of the motor just what it is doing. The ability of a motor to "start on either pole" is easily demonstrated. It is felt that these features make the apparatus useful to the teacher of electricity as well as to the acoustics teacher.

By chalking a radial line on the end of a large motor pulley and using the slit to scan the moving radius, one may observe curves which are the combination of one simple harmonic motion with one circular motion in the same plane. These curves show interesting discontinuities. The individual lines radiate from the origin, the number present depending on the frequency ratio between the two motions. The curves may be conveniently represented by the equation,

$$y = \pm x \tan (2\pi N_y t + \phi_y) \quad \text{where} \quad x = A \sin 2\pi N_x t.$$

Fig. 3 shows the pattern for two diameters at right angles drawn on the pulley end. The pulley is making one rotation to two complete vibrations of the slit. Other curiously distorted stroboscopic effects will doubtless suggest themselves to the reader. Masking the lantern aperture to a fixed slit parallel to the moving one cuts out the distortion, provided the fixed slit be exactly centered. The definition is excellent. The narrowness of the field is no disadvantage when viewing a vibrating string.

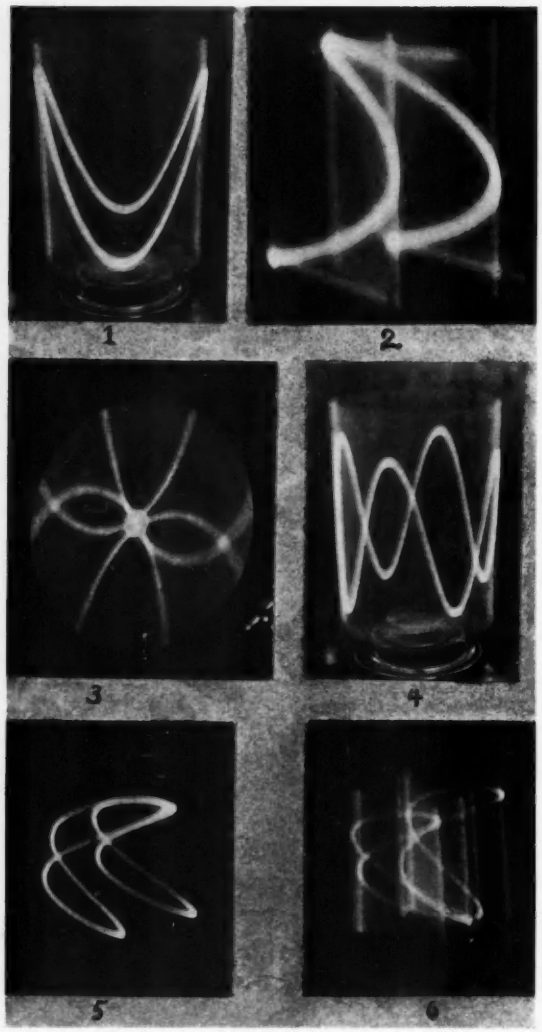

PLATE 1. FIGS. 1–6.

A much more elaborate apparatus than the one described above is shown in the photograph, Plate 2. A different family of curves is obtained, the frequency ratio 1–2, rather than 1–1, being involved. Both ϕ_y and ϕ_z are variable. Double or triple phase changes are possible. The hundredth horsepower motor next to the lantern is a two pole motor. The motor pulley carries an eccentric pin, which in turn is fitted with a very small pulley sheave, over which is looped a strong fishline which runs to the card in the lantern slide position. The string is bound to the card, very near one edge, with metallic cement. It runs beyond the card to a light coil spring

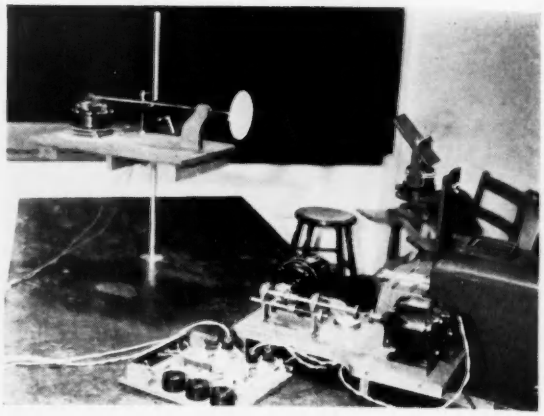

PLATE 2.

which takes up the slack. The spring has a Hooke's constant of 70 grams per centimeter of stretch. Ordinary rubber bands can be used. The card, which runs in a grooved trackway, is $1\frac{1}{2}$ inches wide by $3\frac{1}{2}$ inches long and weighs $1\frac{1}{2}$ grams. The slit in it is 1 inch by 1/32 inch, the slit being at the center of the card at right angles to the fishline. The stroke of the card is 7/8 inch, and, since the fishline is very long in comparison, the motion is a very satisfactory approximation to simple harmonic motion. The string may be removed from the two pole motor and attached to the variable speed shaft, which is driven by the second motor, a four pole. The adjusting screw on this shaft bears forty threads to the inch, to permit minute adjustment of speed. The second motor carries a rear axle shaft extension, fitted with eccentric pin and sheave, to drive a second slotted card which crosses the first at right angles. The spot of light which passes through the crossed slits is focused on the little cardboard screen which shows as a white oval in the picture. With motors running, the complete family of Lissajous' curves appear, z being equal to zero in the general equations given above.

The small screen may be driven by the third motor so as to vibrate toward and away from the lantern. Complicated space curves are seen. Fig. 5 shows the 4–2–1 curve. The lowest sharp bend in the figure is nearest the eye. The next lowest bend is to the rear. The highest portion is at a middle depth. The curve lies on a surface whose cross section is an inverted U. Fig. 2

shows the same curve in a different phase, together with two of the three surfaces on which it lies. The U-shaped surface, extending in the x direction, intersects the W-shaped surface, extending in the y direction, to give the figure. The reader may be able to visualize the third surface, not shown because of photographic difficulties. It is also U-shaped, extends in the z direction, and passes through the intersection of the other two surfaces. Fig. 6 shows the change occurring in the W-shaped surface and in the figure, as one of the motors creeps with respect to the other two.

The motor which drives the screen is an eight pole motor. A manually adjustable stator is a valuable feature of this motor. Another valuable feature is its frequent failure to run synchronously, so that a triple phase creep occurs. The screen is braced at the back with triangular pieces of cardboard, bound with metallic cement. The slow speed motor is a necessity, as windage for the screen is a troublesome problem. The crank arm is wooden with inset metal bearings which are very light in design. Persons in need of additional specifications are asked to communicate with the author.

The following statements may be made concerning the general nature of three-dimensional harmonic curves:

If amplitude factors be taken equal, the curves lie within a cubic space.

The only straight line figure is the 1–1–1 curve having both phase angles zero. The figure is the diagonal of the cubic space.

Curves involving a 1–1 ratio for which the phase angle is zero, lie in a plane which passes through diagonally opposite edges of the cubic space.

The remaining curves are true skew curves.

Excluding intervals of less than the minor third, that is, frequency ratio integers above six, there are forty-two curves existent. (Ratio integer permutations and ratio multiples, which give rise to mirror images and the like, are disregarded. The apparatus fails to show nine of these forty-two curves.)

The projections of the space curves on the xy, xz and yz planes, respectively, are Lissajous' curves.

The space curves lie at the mutual intersection of three mutually perpendicular surfaces, any cross section of these surfaces being a Lissajous curve.

Double phase creep causes a flow of the figure around one of these surfaces, the form of this surface remaining fixed, while the forms of the other two surfaces change.

Triple phase creep results in a change in form for all three surfaces, two of the rates of change being independent, possibly incommensurable, so that the figure does not return to its original form.

The cross sections of the surfaces exhibit the same cyclic changes as do Lissajous' curves.

Single phase creep is impossible.

It is hoped that apparatus for obtaining these curves acoustically will be developed. Complex component motions would lead to figures of almost unbelievable intricacy.

Organ Pipes and Vowel Quality

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(Received November 22, 1934)

RUSSELL[1] describes an experiment as follows:

> The author has a lead-tin, round-walled open organ pipe, pitched at D♯$_4$ (615 d.v./sec.) which gives what is probably as passable an imitation of the vowel *a* (ah) as can be had by such mechanical means. If while this pipe is sounding, it is tightly grasped just above the exit (letting it fall into the soft crotch between the thumb and forefinger which are then pressed against it laterally) it speaks *a u—a u—a u* as plainly as any other mechanical vowels the author has heard. At the same time its pitch is not lowered more than a fraction of a semi-tone. . . . The vowel quality change must therefore have been due to changed partials. . . . Since the *u* (oo) element sounds more muffled or dead, we must conclude without more physical evidence to the contrary that the *mere pinching of* the *pipe walls* at that point *by* the *soft fingers* served to deaden the higher partials; and that as a result of the suppression of those high partials, the vowel quality the pipe conveyed to our hearing was changed from *a* (ah) to *u* (oo). This change was without any doubt due to the effect of wall surfaces, and could in no manner be attributed to any influence whatever of the air volume or capacity of the cavity.

The italics in this quotation are Russell's.

On attempting to repeat Russell's observation with a somewhat similar pipe I found apparently the same phenomenon by a different method. If the mouth is simply shaded by bringing a hand over it the change in vowel is immediately heard. If the pipe is blown steadily while the hand alternately leaves it free and then shades it there is a clear *a u—a u—a u*. Moreover it is not necessary that the object which shades the pipe be soft—a piece of wood can produce the same effect.

I have extended the observations to five pitches on each of two stops in an organ. Both stops consist of open pipes. One is a flute and the other a violoncello. The pipes are so located as to make it difficult to shade the mouths. It proved however entirely satisfactory to shade the open end of the flute, and the open end and tuning slit of the violoncello. The shading was done by hand, and the results on the two stops are much the same.

For e_2, about 163 cycles/sec., the effect is not very good. When the pipe is not shaded it does not give a good *ah*.

For b_2, about 244 cycles/sec., the effect is better, although the *oo* is perhaps as much like the vowel in c*a*ll or c*u*ll as like that in c*oo*l.

For e_3, about 326 cycles/sec., the *oo* is better, and the *oo ah* is rather good.

For e_4, about 652 cycles/sec., the effect is good.

For e_5, about 1304 cycles/sec., the pipe gives *ah* when shaded, and when free a vowel more like that in mach*i*ne. In this case the alternate shading and freeing of the pipe gives an effect more like a repetition of the English long *i* (*ah ee*).

The Physics Department at Smith College has a set of eight wooden stopped diapason pipes which run from middle *c* to the octave above. On these I find that alternately shading the mouth and leaving it free gives a good *oo ah* for the higher pitched pipes, but that on the lower ones the *oo* is not very satisfactory—it is more like the vowel in c*u*p.

The explanation of these phenomena seems to depend on three well-known facts: First, that the frequencies of the normal modes of vibration of the air in an organ pipe do not have precisely the harmonic ratios $1:2:3:4\cdots$; second, that when a pipe is blown steadily the frequencies of the partial tones are forced to assume these harmonic ratios; and third, that shading a pipe throws the normal modes of vibration farther from the harmonic series, so that when a pipe is blown steadily the higher components are made fainter by shading it.

The frequencies characteristic of the vowels in question may be taken to be roughly as indicated below.[2]

	Miller	Collected by Fletcher
p*oo*l	326	400 (and 800)
f*a*ther	1050	825 and 1200
mach*i*ne	308 and 3100	373 and 2400

[1] G. Oscar Russell, *Speech and Voice*, pp. 67–69 (1931).

[2] Dayton Clarence Miller, *The Science of Musical Sounds*, pp. 226–227 (1916). Harvey Fletcher, *Speech and Hearing*, p. 58 (1929).

If the octave is prominent when the pipe is not shaded, and is much weakened when the pipe is shaded, a comparison of these numbers with the frequencies of the pipes in question gives at once an explanation for the *ah ee* from the e_5 pipe, and of something approaching *ah oo* from the e_4. For the e_3 pipe the effect should be fairly good, as it was in fact found to be. For pipes of lower pitch it is necessary to consider components higher than the octave, and for e_2 the component characteristic of *ah* would be of so high an order as to explain the failure of e_2 to give any good *ah*, even when not shaded.

It is well known that it is not always easy to decide just what vowel is heard, but in a general way the explanation of the change of vowel on shading an organ pipe appears to be given by the weakening which the shading produces in the higher components of the sound from the pipe.

Acoustical Society News

For the first time in its history, the Acoustical Society met in affiliation with the A.A.A.S. at Pittsburgh, December 28 and 29. A joint meeting was arranged between the Acoustical Society, Section B (physics) and the American Physical Society. The program included a Symposium on Noise and two invited papers. One of the invited papers was by Mr. B. G. Churcher of the Metropolitan-Vickers Company, Manchester, England, on "A Loudness Scale for Industrial Noise Measurements." He showed that any loudness scale must be decided subjectively by hearing observations but should be expressed in energy units. Other speakers discussing this subject were Dr. E. J. Abbott, Dr. L. B. Ham and Mr. V. L. Chrisler. An important paper was delivered by Dr. V. O. Knudsen on "The Absorption of Sound in Gases," which described how methods employed in acoustics were cleverly adapted to give important and surprising results in the domain of molecular interactions. It is a matter of pride for the Acoustical Society that this paper was awarded the $1000 prize for the noteworthy paper presented at the Christmas meeting of the American Association for the Advancement of Science.

The second invited paper was given by Dr. Hallowell Davis of Harvard University on "The Electrical Phenomena of the Cochlea and the Auditory Nerve." His experiments indicated that the cochlea always responds to mechanical sound impulses, but that the auditory nerve is intermittent in its action,—discharging electrical impulses along the nerves.

An ingenious demonstration by W. C. Dod of Miami University made visible three simple harmonic motions mutually at right angles. Dr. E. C. Wente of the Bell Telephone Laboratories described the various types of instruments available for measuring sound.

Attendance at the meetings varied from fifty to two hundred. Mr. C. R. Hanna of the Westinghouse Company was chairman of arrangements and program. A banquet Friday evening was attended by some fifty enthusiastic participants.

The next meeting of the Society will be held in New York City, April 29–30.

Book Reviews

Bessel Functions for Engineers. N. W. McLachlan. Pp. 192+xii, Fourth Volume of the Oxford Engineering Science Series, Oxford University Press, 1934. Price $5.00.

Although the mathematical genesis of the functions is sufficiently set forth, the outstanding features of this text are the thorough treatment of the practical applications, and the inclusion of numerous examples, many of which develop important side topics. The course of numerical calculations is made quite clear. All functions introduced are graphed, and detailed accounts are given of such matters as the determination of the branch of a function when the argument is complex. A large part of the text develops the applications to important problems in physics and engineering, chiefly in electric circuit theory, vibration of membranes and the author's specialty, sound radiation from loudspeaker horns. Other applications are found in the examples.

The book takes up a remarkable number of topics, though in some cases necessarily rather briefly. There are chapters on the functions of zero and higher orders, expansion in terms of Bessel functions as applied to integration, the hypergeometric, gamma, and Struve functions, asymptotic functions, integrals involving Bessel functions, functions of pure imaginary argument, the ber, bei, ker, kei functions, and the calculation of the resistance of conductors to an alternating current. A list of formulas and an extensive set of references are appended, as well as short tables of the principal functions.

The concreteness of the treatment and the interest of the examples should make this book valuable to the student of pure and applied mathematics as well as to the engineer, as sug-

gested by the author in his preface. The author uses the notation of Whittaker and Watson's *Modern Analysis* and Watson's *Bessel Functions*, so that these treatises can easily be used as references for more complete information on subjects touched upon in the text.

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Modern Acoustics. A. H. DAVIS. Pp. 345. The Macmillan Company, 1934. Price $6.00.

In the Preface to his book, Dr. Davis states that the subject of acoustics has undergone a considerable change in the last twenty years—not so much in the fundamental theory as in the introduction of electrical apparatus and methods. Consequently Rayleigh's *Theory of Sound* and Lamb's *Dynamical Theory of Sound* remain of first importance as rigorous presentations of the theoretical bases of the subject. There has been some adaptation of ideas, particularly *impedance*, which the author uses extensively in his treatment of mechanical vibrations, electrical apparatus for generating and measuring sound, acoustic impedance and sound transmission.

It is interesting to note that most of the many references given came in the past ten years and indicate the activity in acoustics during that time. As might be expected because of Dr. Davis' extended investigations at the British National Physical Laboratory, he discusses in detail the subjects of intensity, absorption and transmission of sound, with the associated theory and methods of measurement. Discussions are also given of practical applications of acoustics—the ear and hearing, the measurement and suppression of noise, acoustics of buildings and the recording and reproduction of sound, etc.

The book is divided into eighteen chapters, with nine appendices, the latter dealing with various mathematical formulae and theoretical treatments of electrical problems. There are 102 figures that help make the reading easier; also various tables and convenient collections of formulae, e.g., Table IV that sets forth the "Intensity Relations for Plane Waves in Air," with numerical values for pressure, energy, etc.

Dr. Davis has worked for years on practical problems in acoustics and his book presents his working knowledge of the subject, the necessary theory and equations to explain the various phenomena and the operation of acoustical instruments; while the results set forth are either his own data or information about which he has a competent opinion. The publication should therefore fulfill the author's expectation that "the book may be of value to students, to technical or research workers in acoustics and to those in trade or industry who need to acquaint themselves with modern acoustical methods or to apply them."

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Torsional Vibration. W. A. TUPLIN. John Wiley & Sons, Inc., New York, 1934. Price $5.50.

The book contains an elementary discussion of various torsional vibration problems and describes the procedure to be followed in the actual calculations of critical speeds and amplitudes of forced vibrations in various practical cases. The author does not use calculus in most portions of the book and calculates frequencies and amplitudes by using carefully arranged tables, the application of which in practical cases requires only slight mathematical knowledge.

In the first two chapters of the book general theory of vibration of a system with one degree of freedom is discussed and it is shown how an actual shaft can be replaced by an equivalent shaft of a constant diameter. Some recently developed formulae for the equivalent length of crankshafts (Carter formula) are given in the second chapter.

Chapters three, four and five of the book deal with simple problems of free vibrations of two- and three-mass systems. In chapters six and seven the multi-mass systems are discussed. In these chapters a useful method is given for derivation of frequency equations. This method is applied in calculation of a six-mass system. Horner's method of solution of numerical equations is used in solving frequency equations.

Chapter nine contains an elementary discussion of harmonic analysis. The approximate method of analysis due to Professor Sylvanus P. Thompson is described and applied to analysis of torque curve for internal combustion engines.

In chapter ten, forced torsional vibrations without damping are discussed. A numerical example of vibration in a multi-cylinder engine drive is considered with all necessary details. The case of branched systems is also discussed and a numerical example of a marine geared turbine drive is given.

In chapter twelve, critical speeds and forced vibrations at resonance are considered. Various kinds of damping are discussed. In the section dealing with energy absorbed by solid friction, some new experimental data regarding hysteresis losses, obtained by S. F. Dorey,[1] are given. The theory is illustrated by a numerical example in which a calculation of stresses at critical speed in a marine propeller shaft is given.

In chapter thirteen, torsional vibration dampers and absorbers are discussed. The Lanchester damper and the Sandner damper are described and the principle of their action is explained. The theory of vibration absorbers, developed by J. Ormondroyd and J. Den Hartog, is given. An example of calculation of a damped torsional vibration absorber for automobile engines is discussed with all necessary details.

Chapter fourteen deals with forced vibration in geared systems. The importance of a torsionally flexible shaft drive is explained and some examples of design of such a shaft are given.

In chapters fifteen, sixteen and seventeen, the effect of mass of a shaft on the frequency of torsional vibration is considered, various kinds of flexible couplings are discussed, and a method of investigating torsional impact in a multi-mass system is given.

All chapters are illustrated with a sufficient number of numerical examples taken from actual cases so that the book can be very useful to engineers interested in torsional vibration.

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[1] See Proc. Inst. Mech. Eng. **123**, 479 (1932).

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The Problem of Noise. F. C. BARTLETT. Pp. 87, 1934. The Macmillan Company. Price $1.25.

This little volume contains, in a somewhat expanded form, the two Heath Clark lectures which Professor Bartlett delivered in London at the invitation of the National Institute of Industrial Psychology. Professor Bartlett is a psychologist and discusses the subject of noise primarily from the standpoint of psychology, but he also uses the results obtained by physicists.

There are two chapters in the book,—the first chapter dealing mainly with the definitions and measurement of noise, the physiological effects of noise; the perception of complex sounds and the masking of sound. The second chapter discusses the psychological problems of noise, the experimental problems, the effects of noise on people and the methods and possibilities for control of noise.

Professor Bartlett has set forth a brief exposition of the subject, based on the scientific findings, with an impartial discussion of the practical means of controlling the nuisance which will interest both the layman and the scientist.

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Signals and Speech in Electrical Communication. JOHN MILLS. Pp. 281, 1934. Harcourt Brace and Company. Price $2.00.

This book gives an exposition of the general principles of electrical transmission both by wire and by radio. It is intended for the nontechnical reader, and contains sixteen short chapters without mathematics or diagrams. As might be expected from the title, the book describes the telephone and its action, wireless, moving pictures, television and other applications of electrical devices. The vacuum tube because of its far-reaching importance receives an extended discussion. In spite of the nontechnical purpose of the book, the reader finds a constant use of electrical terms,—generator, modulator, frequency, inductance, capacitance, etc., which, however, appeal to many laymen including young people, who know more about electrical devices than one would suppose. The book will be interesting and instructive to both nontechnical and technical readers. The author is an engineer with an intimate knowledge of his subject and also an author with years of experience in scientific exposition.

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